ISOMORPHIC PERMUTED EXPERIMENTAL DESIGNS AND THEIR APPLICATION IN CONJOINT ANALYSIS

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ABSTRACT

The paper deals with experimental designs used in conjoint analysis. The described approach permutes the structure of the underlying fractional experimental design to make multiple different sets of combinations. The resulting experimental designs, suggested to be called *Isomorphic Permuted Experimental Designs (IPED)*, are statistically equivalent to each other while combining diverse sets of the variables and levels into different designs. By facilitating distinctive individual designs (for each respondent) IPEDs reduce a bias caused by some possibly unusually strong performing combinations, allows detection and estimation of interactions among variables as well as identification of pattern-based segments emerging from individual models of utilities. The paper researches the theoretical foundation of the approach, formalizes the methodology for algorithmic implementation and shows a practical example of utilization.

KEYWORDS

Conjoint analysis, experimental design, regression analysis, fractional experimental designs, individual designs, dummy variable regression, incomplete concepts, interactions, pattern-based segmentation.

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INTRODUCTION

Conjoint analysis has become increasingly prevalent as a major approach to studying consumer preferences. Green and Srinivasan (1990a, 1990b), Green and Krieger (1991), Krieger, Green and Wind (2004) demonstrate that conjoint analysis has multiple advantages in quantifying consumer preferences. Conjoint analysis assumes that a product or service can be decomposed into its component *variables* (also called *attributes, silos or categories*) and *levels (elements).* By presenting a series of *profiles (concepts),* which are combinations of levels from different variables, to a number of respondents and finding out which are most preferred, conjoint analysis allows the determination of utilities of each of the levels called the individual utilities (part-worth or impact scores) of levels.

Experimental design, a statistical plan that lays out the combinations of the profile elements, is a foundation of conjoint analysis, as well as several other experimentation-based approaches. Despite substantial research that has been done in the field and presented in many works such as Cattin and Wittink 1982; Carroll and Green 1995; Atkinson and Haines 1996 and Atkinson and Bailey 2001, the field is still active and presents an opportunity for research. Box et al. (2005) compares a range of experimental designs varying in the number of *variables (factors)* (applied to attributes in conjoint analysis), and the number of *levels* (matched to the elements in conjoint analysis) combined into *runs* (*experimental units* or rows of design) (matched to profiles in conjoint analysis). For simplification of terminology, we will use terms 'variables', 'levels' and 'profiles' as applicable to both experimental design and conjoint analysis.

Moskowitz et al. (2005) points to the limitations of some traditional experimental designs methods. First, in such approaches an experimental design is applied to a set of variables only once creating a single design for all respondents (optionally randomized). Second, these approaches in most cases utilize the *complete concept approach* (used in full profile conjoint analysis for example), in which every combination of the levels must have all the variables present (at least one of the levels from each variable). In that case, it is impossible to estimate the absolute utility value of a level. Rather, the utility values are estimated relative to a *reference level* - one of the subjectively selected levels. With the complete concepts approach, one cannot compare the utilities of levels across different variables. Rather, one can only compare the utilities of levels within the same variable.

Moskowitz et al. (2005) and Gofman (2006) summarize the limitations of the existing approaches and argue that they present several severe interlinked statistical problems:

Biased Results. A limited number of distinct prototypes leads to a bias in outcome as levels appear in a limited number of combinations and a few potentially very strong levels might skew the results.

Insufficient variation of combinations also prevents detection and estimation of interactions (pair wise and higher order).

Collinearity. Complete concepts do not allow estimating the true utility value of profile levels due to multicollinearity.

No True Estimate of the Basic Level of Interest. The statistical analyses of such complete concepts require effects model regressions, in which there is no estimate of the additive constant (the basic level of consumer interest), and the requirement that the utilities of the levels in each variable add up to zero.

No True Estimate of the Utility Value of the Individual Levels. The requirement of the constant sum equal to zero means that if a new level is introduced into the study, the utilities of the other levels must be readjusted because they have relative value. This readjustment means that one cannot use the results for databasing (absolute values of the utilities that could not be easily, if at all, compared across the variables or projects).

Moskowitz (1994) suggested a practical approach that permutes the structure of the underlying fractional design to make multiple different sets of combinations, although there is no generalized model / description of the approach which hinders the implementation. Initial steps towards formalization of the process has been described in Moskowitz and Gofman (2005) and Gofman (2006).

This paper further develops the permutation approach to experimental designs. Increasing the variability of designs would improve the reliability of data and in turn would facilitate studying interactions among levels and identifying pattern-based segments in individual models of utilities.

PERMUTING EXPERIMENTAL DESIGNS

An alternative to complete concepts approach is shown in Moskowitz et al. (2005). It is called *incomplete concepts* (or with profiles having *zero conditions*). By arraying the combinations of levels in a specified experimental design with true zero's, i.e., with some combinations entirely missing a variable, the researcher can estimate the absolute values of the utilities. As these designs take longer to be balanced, they require more profiles. This drawback is more than compensated in a majority of cases by the ability of these designs to generate absolute values allowing for comparison between the variables and under specific situations, across the projects.

Let's analyze the approach based on a particular fractional experimental design shown in Table 1 - the Plackett Burman 5-Level screening design (Encyclopedia of Statistical Sciences, 1985). It enables the researcher to investigate up to 5 variables in a profile in conjoint analysis, and up to 4 levels per variable. Note that while the experimental design allows for 5 levels per variable, the fifth level is reserved for "null" or "no level present" (depicted as '0' in Table 1). By allowing for a true "zero condition", the researcher can use the regression analysis to better estimate the contribution of every level to respondent reactions (Moskowitz et al., 2005).

Table 1. An example of an experimental design for 25 profiles, based on the Plackett Burman screening design. x_j^i is a permuted design experimental unit for variable *i* and level *j* (0 means a missing level).

	Design				
N	**Var^1**	**Var^2**	**Var^3**	**Var^4**	**Var^5**
1	x_4^1	x_1^2	x_3^3	x_1^4	x_1^5
2	x_0^1	x_4^2	x_1^3	x_3^4	x_1^5
3	x_3^1	x_0^2	x_4^3	x_1^4	x_3^5
4	x_3^1	x_3^2	x_0^3	x_4^4	x_1^5
5	x_2^1	x_3^2	x_3^3	x_0^4	x_4^5
6	x_3^1	x_2^2	x_3^3	x_3^4	x_0^5
7	x_4^1	x_3^2	x_2^3	x_3^4	x_3^5
8	x_1^1	x_4^2	x_3^3	x_2^4	x_3^5
9	x_2^1	x_1^2	x_4^3	x_3^4	x_2^5
10	x_2^1	x_2^2	x_1^3	x_4^4	x_3^5
11	x_0^1	x_2^2	x_2^3	x_1^4	x_4^5
12	x_2^1	x_0^2	x_2^3	x_2^4	x_1^5
13	x_4^1	x_2^2	x_0^3	x_2^4	x_2^5
	Design (cont.)				
N	**Var^1**	**Var^2**	**Var^3**	**Var^4**	**Var^5**
14	x_3^1	x_4^2	x_2^3	x_0^4	x_2^5
15	x_0^1	x_3^2	x_4^3	x_2^4	x_0^5
16	x_0^1	x_0^2	x_3^3	x_4^4	x_2^5
17	x_1^1	x_0^2	x_0^3	x_3^4	x_4^5
18	x_0^1	x_1^2	x_0^3	x_0^4	x_3^5
19	x_4^1	x_0^2	x_1^3	x_0^4	x_0^5
20	x_2^1	x_4^2	x_0^3	x_1^4	x_0^5
21	x_1^1	x_2^2	x_4^3	x_0^4	x_1^5
22	x_1^1	x_1^2	x_2^3	x_4^4	x_0^5
23	x_3^1	x_1^2	x_1^3	x_2^4	x_4^5
24	x_1^1	x_3^2	x_1^3	x_1^4	x_2^5
25	x_4^1	x_4^2	x_4^3	x_4^4	x_4^5

A conceptual model of the traditional approaches to experiential design is shown in Figure 1 (left) in which an experimental design is applied to a set of variables only once creating a single design for all respondents. Optionally, this design is randomized (the same design with limited number of concepts, just the order of the concepts is randomized). A drawback of this approach is that it tests the levels in a limited number of combinations thus preventing detection of all interactions. There is also a possibility of a bias introduced due to the limited and fixed number of concepts tested by all the respondents (Moskowitz et al., 2005).

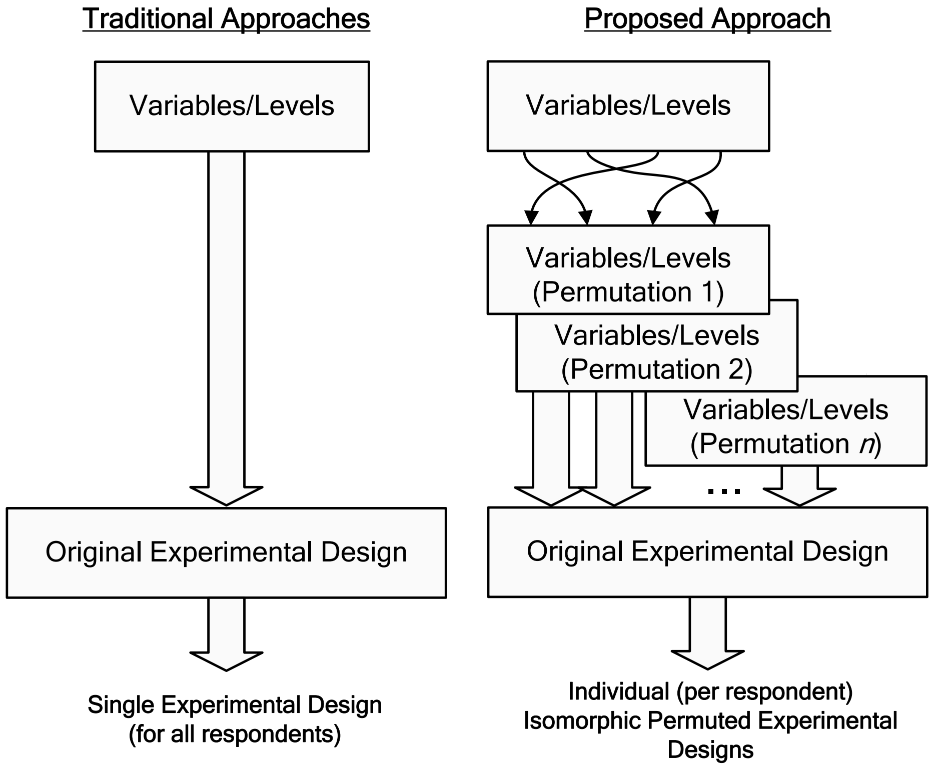


Figure 1. Conceptual model of traditional experimental designs approaches (on the left) and proposed individual isomorphic permuted experimental designs (IPED) (on the right).

If we randomize the sequence of the variables and levels inside the variables *before* applying the experimental design, we can theoretically create a large number of unique concepts comprising unique individual designs (up to statistical limits imposed by the specific design). The proposed approach creates *isomorphic designs* that are statistically equivalent to each other while combining different sets of the levels in different combinations. The methodology is suggested to be called *Isomorphic Permuted Experimental Designs (IPED).* The proposed conceptual model of IPED (Figure 1, right) shows multiple permutations of the variables/levels *before* the experimental design is applied thus creating distinct designs for each respondent with thousands of unique concepts tested (depending on the number of the levels and the sample size). This in turn creates a diverse contextual environment for concept testing producing less biased results (Moskowitz et al., 2005). In addition, it creates a database of information which could be used for identification of the interactions between the levels in the design as it evenly tests all pairwise and higher order interactions with a sufficient sample (Gofman, 2006).

In an experimental design with variables $(A_1, A_2, \ldots, A_n)$, an experimental unit c_i (line i of the design) could be described as shown in Figure 2. A randomized subset $R_k \subseteq (A_1, A_2, \ldots, A_n)$ is selected from the original set of variables A. R_k contains variable placeholders (*virtual variables*). For example, the first level of R_k (R_k^1) might contain variable A_4 from the original set of variables; R_k^2 might contain A_1, etc. In another randomization, the placeholders will contain different variables. Then, the experimental units are applied to the randomized (virtual) set of variables to create individual designs for respondents containing the levels $e_{x(h,ci)}^{R(h,k)}$, where $R(\boldsymbol{h}, k)$ is the actual variable assigned to the placeholder *h* during randomization *k*; $x(\boldsymbol{h}, ci)$ is the level number in the variable $R(\boldsymbol{h}, k)$ corresponding to the experimental unit c_i.

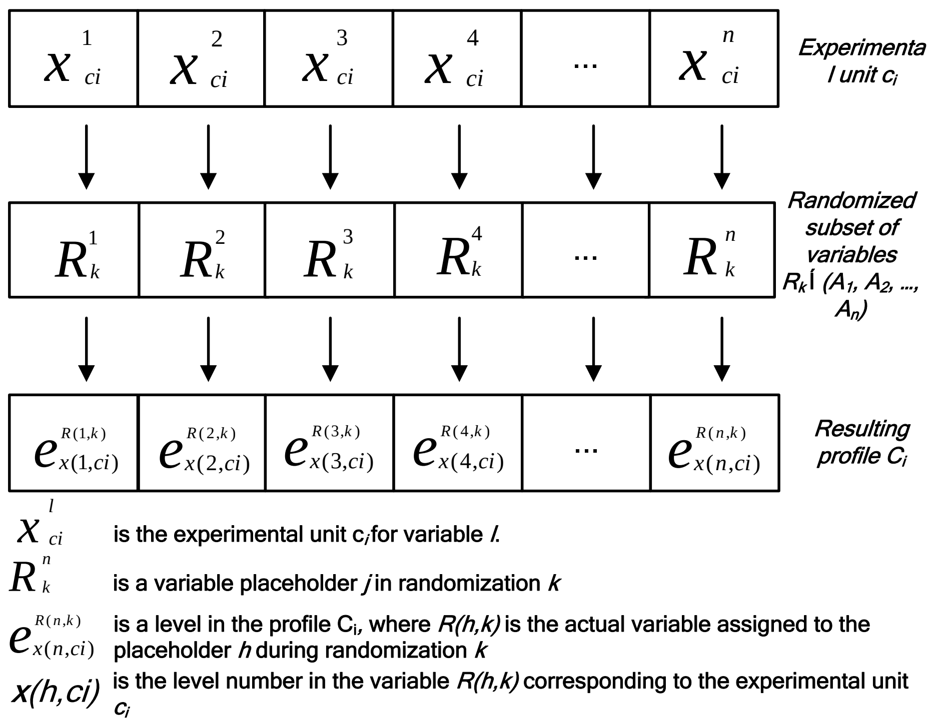


Figure 2. Creation of isomorphic permuted experimental designs. x_{ci}^l is the experimental unit c_i for variable *l*. With a set of variables ($A_1, A_2, …, A_n$), the experimental units are applied to an individually randomized array of variables $R_k ⊆ (A_1, A_2, …, A_n)$, where R_k^j is a variable placeholder *j* in randomization *k*; $e_{x(h,ci)}^{R(h,k)}$ is an level in the profile C_i, where *R(h,k)* is the actual variable assigned to the placeholder *h* during randomization *k*; *x(h,ci)* is the level number in the variable *R(h,k)* corresponding to the experimental unit c_i (shown for the experimental design with matching number of the variables in the design and in the conjoint analysis).

Figure 3 shows an example of an application of an experimental unit to randomized sets of variables. A single experimental unit is shown (line 1 in Table 1). In Set 1, the original order of the variables (A, B, C, D, and E) is used thus creating the profile

$$C_i^1 = \{A_4 \mid B_1 \mid C_3 \mid D_1 \mid E_1\},$$

where C_i^1 is profile *i* for set 1; X_j is level *j* of variable *X* ($X ⊆ (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E})$). The same process is applied to the set of the variables for the rest of the experimental units of the current design.

For Set 2, the order of the categories is different (B, E, C, A and D). The same experimental unit applied to the new randomization (virtual variables) would produce another profile:

$$C_{\downarrow}i^{\uparrow}2 = \{B_{\downarrow}4 \,|\, E_{\downarrow}1 \,|\, C_{\downarrow}3 \,|\, A_{\downarrow}1 \,|\, D_{\downarrow}1\} \;.$$

Note that the level numbers are the same as for C_i^1 although the variables are different.

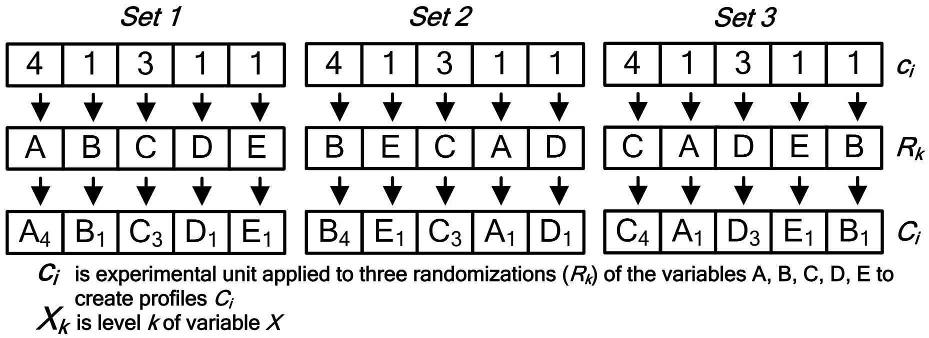


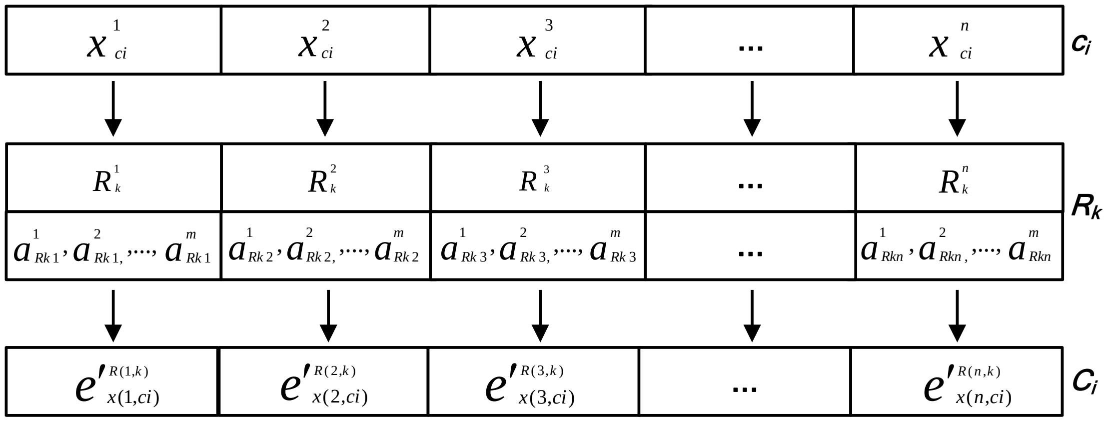
Figure 3. Examples of the creation of isomorphic permuted experimental designs. The experimental unit c_i is applied to three randomizations (R_k) of the variables (A, B, C, D, E) to create concepts C_i. X_j is level j of variable X ($X \subseteq (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E})$. In Set 1, the variables are in the original order. In Sets 2 and 3, the variables are randomized.

For Set 3, the resulting profile is:

$$C_{\downarrow}i^{\uparrow}3 = \{C_{\downarrow}4 \,|\, A_{\downarrow}1 \,|\, D_{\downarrow}3 \,|\, E_{\downarrow}1 \,|\, B_{\downarrow}1\} \;.$$

To enhance the process further by making combinations of the levels more evenly distributed, the idea of the virtual variables could be augmented with the notion of *virtual levels* (placeholder levels). For each randomization, in addition to randomizing the variables order, the levels order in each variable could be randomized as well, as shown at Figure 4.

Figure 4 shows the case of the experimental design with a matching number of variables between the underlying experimental design and conjoint analysis. x_{ci}^{l} is the experimental unit c_i for variable *l*. With a set of variables ($A_1, A_2, \ldots, A_n$), the experimental units are applied to an individually randomized array of variables $R_k \subseteq (A_1, A_2, \ldots, A_n)$. A variable placeholder *j* (R_k^j) in the randomization *k* contains an actual variable while level placeholders a_v^u (level placeholder *v* of the virtual variable *u* contained in the placeholder R_k^j) contain actual levels' numbers. $e'^{R(h,k)}_{x(h,ci)}$ is an level in the profile C_i, where *R(h,k)* is the actual variable assigned to the placeholder *h* during randomization *k*; *x(h,ci)* is the actual level number in the variable placeholder *R(h,k)* with levels placeholders $a_{R(h,k)}^{\boldsymbol{h}}$ corresponding to the experimental unit c_i .



C_i is an experimental unit

x_{ci}^{n} is value of variable *n* in experimental unit c_i

R_k an individually randomized array of variables

R_k^j is variable placeholder *j* in randomization *k*

a_v^u is a level *v* of the actual variable *u* contained in the placeholder R_k^j

$e'^{R(n,k)}_{x(n,ci)}$ is a level in profile C_i, where *R(h,k)* is the actual variable assigned to the placeholder *h* during randomization *k*

Figure 4. Creation of augmented isomorphic permuted experimental designs that includes permutation of the levels and the variables. x_{ci}^{l} is the experimental unit c_i for variable *l*. With a set of variables ($A_1, A_2, \ldots, A_n$), the experimental units are applied to an individually randomized array of variables $R_k \subseteq (A_1, A_2, \ldots, A_n)$, where R_k^j is a variable placeholder *j* in randomization *k;* a_v^u is an level *v* of the actual variable *u* contained in the placeholder R_k^j; $e'^{R(h,k)}_{x(h,ci)}$ is an level in the profile C_i, where *R(h,k)* is the actual variable assigned to the placeholder *h* during randomization *k*; *x(h,ci)* is the actual level number in the variable placeholder *R(h,k)* with levels placeholders $a_{R(h,k)}^{h}$ corresponding to the experimental unit c_i (shown for the experimental design with matching number of the variables in the design and conjoint analysis).

Figure 5 shows examples of the augmented IPED. It uses the same experimental unit as in the example above. In Set 1, the original order of the variables (A, B, C, D, and E) as well as the original order of the levels are utilized thus creating the profile:

$$C_i^{1} = \{A_4 \mid B_1 \mid C_3 \mid D_1 \mid E_1\},$$

where C_i^1 is profile *i* for Set 1; X_j is level *j* of variable *X* ($X \subseteq (\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E})$). The same process is applied to the set of the variables for the rest of the experimental units in the current design.

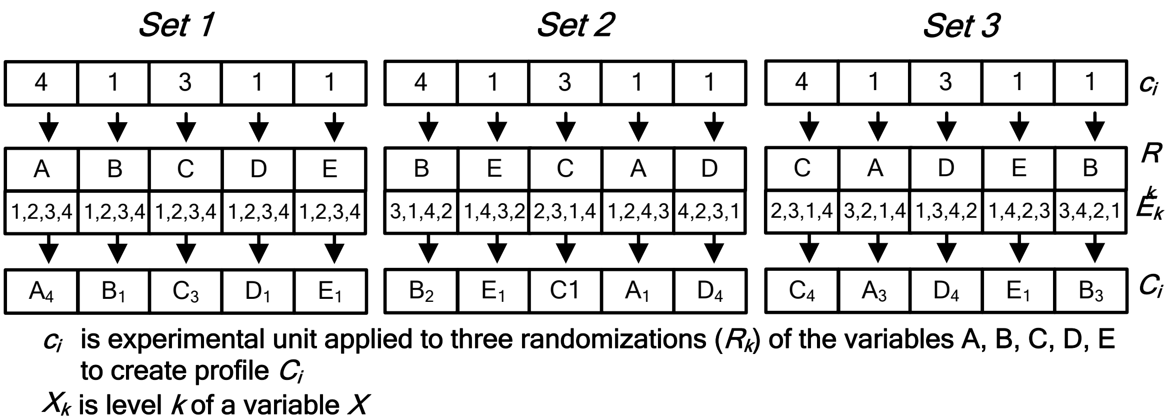


Figure 5. Examples of the creation of augmented isomorphic permuted experimental designs that include permutations of both variables and levels. The experimental unit c_i is applied to three randomizations (R_k) of the variables (A, B, C, D, E) to create profiles C_i. X_j is level j of variable X ($X \subseteq (A, B, C, D, E)$) . Each variable contains four levels (E_k). In Set 1, the variables and the levels are in the original order. In Sets 2 and 3, the variables and levels are randomized.

For Set 2, not only is the order of the categories different (B, E, C, A and D) but the levels are also randomized in each variable. The same experimental unit applied to the new set (virtual variables and levels) would produce another profile:

$$C_i^2 = \{B_2 \,|\, E_1 \,|\, C_1 \,|\, A_1 \,|\, D_4\} \; .$$

For Set 3, the resulting profile is:

$$C_i^3 = \{C_4 \,|\, A_3 \,|\, D_4 \,|\, E_1 \,|\, B_3\} \; .$$

Further throughout the text the term IPED is applied to the augmented IPEDs.

If the number of levels exceeds the capacity of the design selected, it is possible to apply the same design iteratively to parts of the variables and levels thus allowing for testing of different configurations of the variables and levels. To achieve this, two new data objects are introduced to the model: *variable utilization array* W_c:

$$W_c \equiv (w_1, w_2, \ldots, w_n)$$

and *levels utilization matrix* M_e:

$$M_e \equiv \begin{bmatrix} e_1^1 & e_1^2 & e_1^3 & \ldots & e_1^m \\ e_2^1 & e_2^2 & e_2^3 & \ldots & e_2^m \\ e_3^1 & e_3^2 & e_3^3 & \ldots & e_3^m \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ e_n^1 & e_n^2 & e_n^3 & \ldots & e_n^m \end{bmatrix} ,$$

where w_i is variable utilization frequency (how many times variable i has been selected in the permutations); e_i^j is levels utilization frequency (it keeps the number of times level j of variable i has been utilized); $m = \max_i A_i$ – the largest number of the levels in the variables $A_1, A_2, \ldots A_n$ (the empty cells are padded with zeros up to the maximum size of m).

In the first iteration, a random subset R_1 of variables A is chosen: $R_1 \subseteq A$. The elements of the array W_c corresponding to the selected variables are adjusted (incremented to reflect the utilization of the variable in the iteration). This means that if variable A_j has been selected for the current iteration (permutation) of the experimental design, then $w_j = w_j + 1$. After all the experimental units of the current iteration have been used, a new selection of the variables is executed:

$$R_2 \subseteq A,$$

where $R_1 \neq R_2$ if the structure of the design does not exactly match the size of the project.

This process continues until all variables are tested. During the selection process for R_2 (and following iterations if required by a specific set of variables), array W_c is checked to balance utilization of the variables. In the ideal case, every variable should be selected an equal number of times across multiple iterations of the experimental design. In reality, the process aims to minimize the standard deviation of the number of tested variables contained in the array W_c:

$$\sigma_c = \min \sqrt{\frac{1}{\mathbf{n}\left[\sum_{i=1}^{n}(w_i - \overline{w})\right]}^{2}}$$

where $\overline{w}$ is the mean of variable utilization array levels.

For practical purposes such as simplification of algorithmic implementation, the previous expression could be simplified as the following:

$$\varsigma_c = \min \sum_{i=1}^{n} (w_i - \overline{w})^2$$

where ς_c is a minimization criterion for selecting variables in IPED.

A similar approach is applied when the number of levels in the variables exceeds the number of the levels in the experimental design. In the first iteration, a random subset of levels E_1^i in each variable i is chosen:

$$E_1^i \subseteq A_i ,$$

where A_i is a full set of the levels in variable i.

The elements of the matrix M_e corresponding to E_1^i are adjusted (incremented to reflect the utilization of the levels in the profile). This means that if the level j in variable A_i (a_i^j) has been selected for the current iteration of the experimental design to the array E_1^i, then $e_i^j = e_i^j + 1$. After all the experimental units of the current iteration have been used, a new selection of the variables is executed:

$$E_2^i \subseteq A_i,$$

where $E_1^i \neq E_2^i$.

This process continues until all levels are tested. During the selection process for E_2^i (and following iterations if required by the specific set of levels), matrix M_c is used to balance utilization of the levels. In the ideal case, every level should be selected an equal number of times across multiple iterations of the experimental design. It might be difficult or even impossible to achieve that if the variables have unequal sizes. In that case, the process aims to minimize standard deviation of the tested levels distribution for each row of the matrix M_c (levels of the same variable). For the row *c* (variable *c*):

$$\sigma_e^c = min\sqrt{\frac{1}{m_c}\sum_{i=1}^{mc}(e_c^i - \overline{e_c})^2}$$

with parsimoniously simplified expression for implementation:

(↓e↑c = min∑↓(i = 1)↑mc≣ 〖(e↓c↑i − (e↓c)¯)〗 ↑2

where (↓e↑c is a minimization criterion of selecting levels into IPED; $\overline{e_c}$ is mean of the row *c* of matrix levels utilization M_e; m_c is the number of levels in variable *c*.

Furthermore, if the levels have some constraints (mutual restriction, e.g., cannot appear together due to semantic or technological incompatibility), a new data object, *levels constrains matrix,* is introduced:

&C_1^1& ⋯ & C_{n1}^1&C_1^2 & ⋯ &C_{n2}^2& ⋯ &C_1^k& ⋯ &C_{nk}^k@&0&0&0&r_{11}^{21}& ⋯ &r_{11}^{2n2}& ⋯ &r_{11}^{k1}& ⋯ &r_{11}^{knk}@&0&0&0& ⋯ & ⋯ &

where C_j^i is the level *j* of the variable *i* (the first row and the first column, shaded, are shown as captions); r_{xy}^{uv} is a restriction between the level *v* of the variable *u* and the level *y* of the variable *x*.

In most practical cases, this matrix is either empty (no constraints) or sparsely populated. During the process of level selection the levels constrains matrix is checked to each new levels (and if needed variables) selection and the randomization is performed until the constrains are satisfied. The levels in the same variable are always restricted each to another as no two levels from a variable could appear together in one profile.

It is possible to accommodate variable constraints as well by introducing a *variables constraints matrix*:

	C_1	C_2	⋯	C_n
C_1	0	v_1^2	⋯	v_1^n
C_2	v_2^1	0	⋯	v_2^n
⋯	⋯	⋯	0	⋯
C_n	v_n^1	v_2^2	⋯	0

where C_i is the variable *i*; v_j^i is the restriction between variables *i* and *j*.

If two variables are restricted, they can't appear together. This means that the variable selection step should be repeated until the unrestricted variables are selected. Of course, the number of variables in the project should be larger than the number of the variable in the experimental design. Otherwise, any restriction would fail the project.

A DEMONSTRATION OF THE APPROACH

Following the steps described in Gofman and Moskowitz (2009), which analyzes a study of consumer preferences in a mature food category, we will demonstrate the approach on the example of donuts concepts. For demo purposes, sensory, image, usage and other descriptions of donuts are structured as a set of four variables with three levels in each (Table 2). The study utilizes the Plackett Burman 4-Factor 4-Level screening design (Encyclopedia of Statistical Sciences, 1985) with one level in each factor reserved for 'zero condition'. This fractional design requires 20 profiles for each respondent (Table 3). Here, x_0^i represents 'zero condition' for category *i* when the category is not present in the test profile. Each level x_j^i (experimental unit for factor *i*, variable *j*) is applied to a set of variables and levels from a specific project. In our case, it would be an individually permuted selection of variables and levels. The process results in individual experimental designs for each respondent that are unique (to statistically possible) yet isomorphic. Table 4 shows examples of two permuted individual experimental designs for two respondents in a form prepared for dummy variable regression. In this table, levels of the study e_j^i (level *j* of variable *i*) have value '1' if they are present in concepts (rows) and '0' if they are absent. Together, the levels are independent variables in regression with the rating serving as the dependent variable (the last column in Table 4).

Table 2. Variables and levels of the sample project

Code	**ELEMENTS**
	VARIABLE A: BENEFIT
A1	Simply the best cinnamon rolls in the whole wide world
A2	Made fresh … especially for you … by you
A3	From your favorite local bakery or pastry shop
	VARIABLE B: EMOTIONAL
B1	A joy for your senses.. seeing, smelling, tasting
B2	It feeds THE HUNGER
B3	When you think about it, you have to have it… and after you have it, you can't stop eating it
	VARIABLE C: PRIMARY ATTRIBUTE
C1	Big, 3 inch spiraled rounds of dense chewy pastry like a donut with sweet cinnamon inside, covered with sweet icing
C2	Huge, thick, 4 inch spiraled rounds of light flaky pastry with sweet cinnamon inside, covered in a cream cheese frosting
C3	The ultimate chocolate indulgence with rich chocolate inside a huge, thick and gooey spiraled cinnamon bun with sweet icing and a gooey chocolate dripping over the top
	VARIABLE D: MOOD
D1	Premium quality… that great classic taste, like it used to be
D2	With extra chocolate, cream cheese, or sugary icing on the side just waiting for dipping
D3	100% natural… and new choices every month to keep you tantalized

Table 3. Example of original (source) experimental design utilized for individual permutations.

Design				
Unit	**Var^1**	**Var^2**	**Var^3**	**Var^4**
1	x_3^1	x_2^2	x_0^3	x_3^4
2	x_3^1	x_0^2	x_0^3	x_2^4
3	x_2^1	x_0^2	x_3^3	x_3^4
4	x_2^1	x_0^2	x_2^3	x_2^4
5	x_0^1	x_3^2	x_3^3	x_3^4
6	x_0^1	x_2^2	x_2^3	x_1^4
7	x_3^1	x_3^2	x_3^3	x_1^4
8	x_0^1	x_2^2	x_1^3	x_1^4
9	x_1^1	x_3^2	x_1^3	x_2^4
10	x_0^1	x_1^2	x_1^3	x_0^4

Design (cont.)				
Unit	**Var^1**	**Var^2**	**Var^3**	**Var^4**
11	x_1^1	x_1^2	x_2^3	x_3^4
12	x_3^1	x_1^2	x_0^3	x_2^4
13	x_1^1	x_2^2	x_3^3	x_0^4
14	x_1^1	x_0^2	x_2^3	x_3^4
15	x_2^1	x_3^2	x_0^3	x_1^4
16	x_2^1	x_2^2	x_3^3	x_2^4
17	x_3^1	x_0^2	x_1^3	x_0^4
18	x_0^1	x_3^2	x_2^3	x_0^4
19	x_2^1	x_1^2	x_0^3	x_0^4
20	x_1^1	x_1^2	x_1^3	x_1^4

Figure 6 shows sample screen captures of the interview in which a respondent evaluated experimentally designed concepts on a 1 to 9 rating scale.

Table 4. Examples of two permuted individual experimental designs (for two respondents) with ratings assigned by respondents to each individual profile. The permuted designs are based on the source design (Table 3).

Unit	Levels												Rating
	e_1^1	e_2^1	e_3^1	e_1^2	e_2^2	e_3^2	e_1^3	e_2^3	e_3^3	e_1^4	e_2^4	e_3^4	
Respondent 1													
1	0	0	0	0	1	0	0	0	1	1	0	0	**9**
2	0	0	1	0	0	1	0	0	0	1	0	0	**5**
3	0	0	1	1	0	0	0	0	1	0	0	0	**5**
4	1	0	0	1	0	0	1	0	0	0	0	0	**5**
5	0	1	0	0	1	0	0	1	0	0	1	0	**5**
6	0	1	0	0	0	0	1	0	0	1	0	0	**5**
7	0	0	1	0	0	0	0	0	1	0	0	1	**5**
8	0	0	0	0	0	0	0	1	0	0	1	0	**9**
9	0	1	0	0	0	0	0	1	0	1	0	0	**9**
10	0	1	0	0	0	1	0	0	1	0	0	1	**7**
11	0	0	1	0	1	0	1	0	0	0	0	0	**5**
12	1	0	0	0	0	1	0	0	0	0	1	0	**2**
13	0	0	0	0	0	1	0	1	0	0	0	0	**2**
14	0	0	1	0	1	0	1	0	0	0	1	0	**5**
15	0	0	0	1	0	0	0	0	0	0	1	0	**7**
16	1	0	0	0	1	0	0	1	0	0	0	1	**2**
17	1	0	0	1	0	0	0	0	1	1	0	0	**2**
18	0	1	0	1	0	0	0	0	0	0	0	1	**6**
19	1	0	0	0	0	1	0	0	0	0	0	0	**3**
20	0	0	0	0	0	0	1	0	0	0	0	1	**9**

Unit	Levels												Rating
	e_1^1	e_2^1	e_3^1	e_1^2	e_2^2	e_3^2	e_1^3	e_2^3	e_3^3	e_1^4	e_2^4	e_3^4	
Respondent 2													
1	1	0	0	0	0	1	0	0	0	0	1	0	**1**
2	0	0	1	1	0	0	0	0	0	0	0	0	**4**
3	0	0	0	1	0	0	0	0	0	1	0	0	**1**
4	0	0	0	0	0	1	1	0	0	0	0	1	**1**
5	0	0	1	0	0	0	0	1	0	0	1	0	**3**
6	0	1	0	0	0	0	0	0	1	0	0	0	**7**
7	0	1	0	0	1	0	1	0	0	0	1	0	**2**
8	0	0	0	0	0	1	1	0	0	1	0	0	**1**
9	0	1	0	1	0	0	0	0	1	0	0	0	**1**
10	0	0	1	0	0	1	0	0	1	0	1	0	**1**
11	1	0	0	1	0	0	0	1	0	0	0	1	**1**
12	0	0	1	0	0	0	0	0	1	0	0	1	**5**
13	1	0	0	0	0	0	0	1	0	0	0	1	**1**
14	0	0	0	0	1	0	0	0	0	0	0	1	**1**
15	0	0	1	0	1	0	1	0	0	0	0	0	**1**
16	1	0	0	1	0	0	1	0	0	1	0	0	**1**
17	0	1	0	0	0	0	0	0	0	1	0	0	**7**
18	0	1	0	0	0	1	0	1	0	0	0	0	**1**
19	0	0	0	0	1	0	0	1	0	0	1	0	**1**
20	1	0	0	0	1	0	0	0	1	1	0	0	**1**

Figure 6. Sample screen captures of a respondent interview (utilizing Ideamap.NET online tool). The first profile has one variable missing.

The detailed results interpretation, pattern based segmentation of consumers and interaction analyses could be found in Gofman and Moskowitz (2009) and in Gofman (2006).

DISCUSSION AND CONCLUSIONS

The permuted designs are pivotal to the ability to detect any and all interactions, introduced in Moskowitz and Gofman (2005) and Gofman (2006). Due to the randomized permutation of the design, IPED could create hundreds of isomorphic executions that are unique (up to statistically possible in a specific design). As a result, a sufficiently large sample of respondents could evenly test every possible pairwise combination (and higher order interactions if needed) of the levels multiple times thus creating an opportunity to analyze their contribution to the additive model through a regression.

The application of IPEDs to individual respondents models provides the following benefits:

- elimination of selection bias
- homogeneous testing of the levels of the conjoint analysis ensuring multiple exposure of each possible combination of the level to the respondents (with a reasonable number of respondents)
- facilitation of detecting and estimation of the interactions between the levels (including the higher order interactions)
- facilitation of segmentation of the respondents based on the patterns of their responses.

The researched approach tests multiple prototypes with unique combinations of levels on an individual basis thus improving the robustness of the predicted consumer preferences data compared to the traditional approaches of evaluating a limited number of pre-selected prototypes or utilizing a single experimental design for every respondent. This results in better products and launches that are more successful.

The nature of isomorphic permuted experimental designs facilitates the discovery of the latent consumer needs and wishes that they might not be able to identify themselves or articulate in an actionable way. Resulting patterns-based segmentation and discovery of any and all interactions lead to the rules (quantitative relationships between the features) that make the products or services more targeted and competitive.

There are many new applications of the approach to the emerging areas such as Web page design and package optimization (Gofman 2007; Gofman, Moskowitz, and Mets 2009a,b). The advantages of the approach, such as the ability to database and compare results, opened doors to a new science Mind Genomics (Moskowitz, Gofman, Beckley and Ashman 2006; Moskowitz, Gofman and Beckley 2006; Gofman, Moskowitz, and Mets 2009c).

Some practical limitations should be accounted for in the applications of the methodology. An average respondent has an attention span of about 15 minutes (Moskowitz et al., 2005) and thus could evaluate up to 60-75 concepts (approximately up to 40 levels). For this size of conjoint analysis project, a sample of 200 or more respondents would produce a dataset for statistically significant analyses of pairwise interactions between the levels. Furthermore, with a larger amount of data it is possible to detect and estimate explicit interactions between every combination of three levels in the projects although the empirical data is not sufficient at the point of writing this paper to decide about the importance of the third and higher levels of interactions.

In the majority of practical applications, the projects are executed without restrictions to avoid the complexity of setting and satisfying the constraints. In addition, any constraint would make the experimental design less robust. In some cases, they should be considered during the analysis stage rather than during the data collection.

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