Quasiparticles in vortex line liquid phase of layered d-wave superconductors

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We carry out a quantum mechanical analysis of the behavior of nodal quasiparticles in the vortex line liquid phase of planar d-wave superconductors. Applying a novel path integral technique we calculate a number of experimentally relevant observables and demonstrate that in the low-field regime the quasiparticle scattering rates deduced from photoemission and thermal transport data can be markedly different from that extracted from tunneling, specific heat, superfluid stiffness or spin-lattice relaxation time.

In recent years, the physics of nodal quasiparticles in planar d-wave superconductors such as the high-T_c cuprates has attracted a lot of both, theoretical and experimental, attention. The spectroscopic and transport properties of these Dirac-like quasiparticles with a linear dispersion have been studied in quite some detail, and elaborate analyses of various mechanisms of elastic scattering in the uniform superconducting state have been carried out, including the effects of potential [1], Kondo-like [2], and extended impurities [3] as well as twin boundaries [4].

In the mixed state, the recent quantum mechanical generalization of the earlier semiclassical approach [5] proposed in Ref. [6] allows one to account for the non-uniformity of the local d-wave order parameter $\Delta_{\mathbf{p}}(\mathbf{r}) = \Delta\cos(2\theta_{\mathbf{p}})\exp(i\phi(\mathbf{r}))$ by means of a singular gauge transformation from the physical electrons $c_\sigma(\mathbf{r})$ of spin σ to the new fermionic quasiparticles. Unlike electrons, the latter are subject to the effective magnetic field with zero mean which, besides the physical field, also includes the supercurrent circulating outside vortex cores.

Applying the gauge transformation of Ref. [6] to the electronic states with energies small compared to the maximum gap one can represent them in terms of the Nambu operators creating the auxiliary fermions with the momenta near the nodes of $\Delta_{\mathbf{p}}(\mathbf{r})$

$$\begin{pmatrix} c_\sigma(\mathbf{r}) \\ c^\dagger_{-\sigma}(\mathbf{r}) \end{pmatrix} = \sum_{n=1,2;\pm} e^{\pm i\mathbf{k}_F^n \mathbf{r}} \begin{pmatrix} e^{i\phi_A(\mathbf{r})} u_{n\sigma}(\mathbf{r}) \\ e^{-i\phi_B(\mathbf{r})} v_{n\sigma}(\mathbf{r}) \end{pmatrix}, \qquad (1)$$

where $n = 1, 2$ labels the pairs of the opposite nodes, while the choice of the phases $\phi_{A,B}(\mathbf{r})$ is only restricted by the condition $\phi_A(\mathbf{r}) + \phi_B(\mathbf{r}) = \phi(\mathbf{r})$.

In the case of a regular vortex lattice, the representation (1) was used to demonstrate that the structure of the quasiparticle energy spectrum is that of the energy bands, rather than the Landau levels [7].

In the present paper, we extend the analysis based on the representation (1) to the experimentally well-documented vortex line liquid (VLL) phase [8] where the vortices are distributed totally randomly due to their strong pinning by columnar or other defects.

Any disorder, including that induced by random vortices, is expected to affect the behavior of the d-wave quasiparticles most strongly at the lowest energies, possibly resulting in the complete quasiparticle localization which, however, is still awaiting for experimental confirmation [9]. In what follows, we focus on the ballistic regime of quasiparticle energies large compared to the localization scale (see below) which is readily accessible by a number of standard probes such as angular-resolved photoemission (ARPES), thermal transport, tunneling, specific heat, muon spin rotation (μSR), and spin-lattice relaxation.

In the quantum-mechanical approach of Ref. [6], the combined effect of external magnetic field $\mathbf{H} = \nabla \times \mathbf{A}(\mathbf{r})$ and swirling supercurrent characterized by the superfluid velocity $\mathbf{v}_s(\mathbf{r}) = \frac{\hbar}{2}(\nabla\phi_A + \nabla\phi_B) - \frac{e}{c}\mathbf{A}$ does not amount solely to the semiclassical Doppler shift $\epsilon \to \epsilon - \mathbf{k}_F\mathbf{v}_s(\mathbf{r})$ of the quasiparticle energies [5]. The latter is to be complemented by the vector potential, $\mathbf{a}(\mathbf{r}) = \frac{\hbar}{2}(\nabla\phi_A - \nabla\phi_B)$, that couples to the quasiparticles via their momentum, $\mathbf{k} \to \mathbf{k} - \mathbf{a}(\mathbf{r})$, and accounts for the quantum mechanical Berry phase corresponding to their Bohm-Aharonov (BA) scattering by the vortices. Thus, the complete Hamiltonian of the non-interacting nodal quasiparticles contains both the scalar- and the vector-like random terms

$$\mathcal{H} = \sum_{n,\sigma} \int d\mathbf{r}\, \overline{\psi}_\sigma [\hat\gamma_0 \mathbf{k}_F^n \mathbf{v}_s(\mathbf{r}) + v_i\hat\gamma_i(p_i - a_i(\mathbf{r}))]\psi_\sigma. \qquad (2)$$

In Eq.(2), we used the 4×4 representation for the matrices $\hat\gamma_\mu = (\hat\sigma_2, i\hat\sigma_1, i\hat\sigma_3)\otimes\hat\sigma_3$ acting in the space of the Dirac bi-spinors composed of the Nambu spinors: $\psi_\sigma = [(u_{1\sigma}, \epsilon_{\sigma\sigma'}v_{1\sigma'}), (u_{2\sigma}, \epsilon_{\sigma\sigma'}v_{2\sigma'})(\hat\sigma_1 + \hat\sigma_3)/\sqrt{2}]$.

In order to keep our discussion and formulas relatively simple we consider the case of isotropic quasiparticle dispersion and use the units where $v_i = hc/2e = k_B = 1$. Moreover, because of the predominantly small-angle nature of the quasiparticle scattering by the vortices we choose to neglect the processes of inter-node scattering. While anticipating that neither of these simplifying assumptions will affect our main conclusions, we recognize that a combination of the above factors in the case of the real cuprates may give rise to an additional one-to-two dimensional crossover regime [10].

As in the previous studies of the VLL phase [11], we average over different vortex configurations by assuming the Gaussian distributions

$$\langle g^i_{(v,a)}(\mathbf{q}) g^j_{(v,a)}(-\mathbf{q})\rangle = w_{(v,a)}(\mathbf{q}) \left(\delta_{ij} - \frac{q_i q_j}{\mathbf{q}^2}\right), \quad (3)$$

for both $\mathbf{g}_v = \mathbf{v}_s$ and $\mathbf{g}_a = \mathbf{a}$. The striking difference between $w_v(\mathbf{q}) = \alpha/(\mathbf{q}^2+\alpha)$ and $w_a(\mathbf{q}) = \alpha/\mathbf{q}^2$ which are both proportional to the areal density of vortices $\alpha = 2\pi H$ reflects the presence of screening for the scalar-like Doppler potential and its absence for the vector-like BA scattering. The longitudinal ($\propto q_i q_j/\mathbf{q}^2$) part of the correlator (3) proves to contribute negligibly to all the quantities of interest (besides, it is suppressed by the Coulomb interactions).

In contrast to the previous analyses which focused solely on the effect of Doppler scattering [11], we find that vortex disorder has a profound effect on the quasiparticle spectrum which can not be adequately modelled by a constant quasiparticle width. In order to illustrate this point, we apply the standard self-consistent Born equation

$$\Sigma(\epsilon, \mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^2} \frac{\epsilon + \Sigma(\epsilon, \mathbf{q})}{\mathbf{q}^2 - (\epsilon + \Sigma(\epsilon, \mathbf{q}))^2} w_{(v,a)}(\mathbf{p}+\mathbf{q}) \quad (4)$$

to the separate contributions of the two scattering mechanisms towards the total quasiparticle width $\mathrm{Im}\Sigma_v + \mathrm{Im}\Sigma_a$.

In the case of scalar disorder Eq.(4) yields $\mathrm{Im}\Sigma_v \propto \alpha^{1/2}$ for small energies and momenta ($\epsilon, p \lesssim \alpha^{1/2}$) while at $\max(\epsilon,p) \gg \alpha^{1/2}$ it behaves as $\propto \alpha/\max(\epsilon,p)$ which dominates over the scattering by the vortex cores whose rate is estimated as $\propto \alpha/\Delta$.

At first sight, the effect of the BA scattering may seem to be much stronger, since a naive solution Σ_a of Eq.(4) with the singular kernel $w_a(\mathbf{q})$ is plagued with a logarithmic infrared divergence of the momentum integral.

In order to avoid this spurious divergence which stems from the non-gauge invariant nature of the auxiliary fermion propagator one has to proceed directly with computing the manifestly gauge-invariant (retarded) Green function of the physical electrons

$$G^R(\epsilon, \mathbf{r}) = \int_0^\infty dt e^{i\epsilon t} \langle c(t,\mathbf{r}) c^\dagger(0,\mathbf{0})\rangle = \sum_{n=1,2;\pm} e^{\pm i\mathbf{k}_F^n \mathbf{r}}$$

$$\langle \psi_n(\mathbf{r}) \exp[-i\int_C (\mathbf{v}_s + \mathbf{a}\,\sigma_3 \otimes \mathbf{1}) d\mathbf{r}'] \overline{\psi}_n(\mathbf{0})\rangle. \quad (5)$$

It turns out that the exponential decay of this function (see below) makes it largely independent of the contour C which can then be chosen as the straight path between the end points $\mathbf{r}$ and $\mathbf{0}$.

To compute the amplitude (5) we apply the path-integral representation of Ref. [12] to the propagator of the auxiliary Dirac fermions. First, for a fixed vortex configuration, we cast Eq.(5) in the form of a functional integral over the space-time coordinate $\mathbf{r}(\tau)$ and the conjugate momentum $\mathbf{p}(\tau)$ parameterized by the proper time τ

$$\mathcal{G}^R(\epsilon, \mathbf{r}\,|\,\mathbf{v}_s, \mathbf{a}) = \int_0^\infty d\tau \int_{\mathbf{r}(0)=0}^{\mathbf{r}(\tau)=\mathbf{r}} D\mathbf{r}\, D\mathbf{p}\, e^{i\hat{S}_0[\tau]}$$

$$\exp[i\int_0^\tau d\tau'(\mathbf{k}_F \mathbf{v}_s + \frac{d\mathbf{r}}{d\tau}\mathbf{a}) - i\int_C (\mathbf{v}_s + \mathbf{a}) d\mathbf{r}')], \quad (6)$$

where we dropped, for the sake of compactness, the sum over the nodal points and introduced the free fermion action

$$\hat{S}_0[\tau] = \int_0^\tau d\tau' [\epsilon\hat{\gamma}_0 + \mathbf{p}(\frac{d\mathbf{r}}{d\tau'} - \hat{\gamma})]. \quad (7)$$

Averaging over the disorder variables $\mathbf{v}_s$ and $\mathbf{a}$ with the use of Eq.(3) results in the electron Green function $G^R(\epsilon, \mathbf{r}) = \langle \mathcal{G}^R(\epsilon, \mathbf{r}|\mathbf{v}_s, \mathbf{a})\rangle$ which is given by Eq.(6) where, instead of the exponential phase factor, the integrand contains a product of two attenuation factors

$$W_{(v,a)}[\mathbf{r}(\tau)] = \exp[-\frac{1}{2}\int \frac{d\mathbf{q}}{(2\pi)^2} \int_0^\tau d\tau_1 \int_0^\tau d\tau_2$$

$$\mathbf{u}_i(\tau_1)\mathbf{u}_i(\tau_2) w_{(v,a)}(\mathbf{q}) e^{i\mathbf{q}(\mathbf{r}(\tau_1)-\mathbf{r}(\tau_2))}], \quad (8)$$

with $\mathbf{u}_v = \mathbf{v}_F - \mathbf{r}/\tau$ and $\mathbf{u}_a = d\mathbf{r}/d\tau - \mathbf{r}/\tau$. Thus, the presence of the exponent of the line interal taken along the contour C in Eq.(5) strongly reduces the effect of both $\mathbf{a}$ and $\mathbf{v}_s$, as compared to the case of the gauge-variant propagator of the auxiliary fermions. This observation seems to have been overlooked in the earlier studies of this and related problems where the phase factor in question would either not appear at all [13] or be averaged separately from the fermion propagator computed in a different approximation [14].

Proceeding along the lines of the previous analyses of the problem of non-relativistic fermions subject to a random vector potential [15], one can show that in the ballistic regime (which in the present case is defined by the condition $\epsilon \gg \alpha^{1/2}$) the path integral (6) is dominated by the fermion trajectories which only slightly depart from the straight line $\mathbf{r}_0(\tau) = \mathbf{v}\tau$. Evaluating the factor W_v for such a trajectory, one obtains

$$W_v[\mathbf{r}(\tau)] \approx \exp\left[-r\left(\sqrt{\frac{\tau}{r}} - \sqrt{\frac{r}{\tau}}\right)^2 \int_0^\infty \frac{dq}{2\pi} w_v(q)\right], \quad (9)$$

while the integral in W_a turns out to be proportional to the so-called Amperian area of the closed contour composed of a fermion trajectory $\mathbf{r}(\tau)$ and the "return" path $-\mathbf{r}_0(\tau)$. Although this purely geometrical term vanishes for the saddle-point trajectory $\mathbf{r}(\tau) = \mathbf{r}_0(\tau)$, its expansion to first order in the transverse deviation $r_\perp(\tau)$ yields

$$W_a[\mathbf{r}(\tau)] \approx \exp[-\frac{\alpha}{2}\int_0^\tau d\tau' |r_\perp(\tau')|]. \quad (10)$$

By analogy with the non-relativistic problem studied in Refs. [15] the path integral (6) with the W_i factors given by Eqs.(9) and (10) can be related to the resolvent of the Schroedinger equation describing the transverse motion of the Dirac fermion

$$\left[\partial_x^2 + (\epsilon^2 - q^2) + (\alpha\,\epsilon\,/q)^2\,(|x| + x_0))^2 + i\alpha\,\mathrm{sign}\,x\right]$$

$$\times g(\epsilon, q|x, x') = \delta(x - x'), \qquad (11)$$

where $x_0 = (\sqrt{|\epsilon/q|} - \sqrt{|q/\epsilon|})^2/2\alpha^{1/2}$.

By analogy with the results of Refs. [15] the averaged physical electron propagator $G^R(\epsilon, \mathbf{p})$ can be obtained by convoluting the kernel $1/(\mathbf{p}^2 - q^2)^{3/2}$ with the solution of (11) taken at $x = x' = 0$ which is given by the formula

$$g(\epsilon, q|0, 0) = \left[\frac{d}{dx}\ln(U_+U_-)\right]^{-1}_{x=0} \qquad (12)$$

where $U_\pm = U\left(a_\pm, \lambda[x_0 \pm x]\right)$ is the parabolic cylinder function of the parameter $a_\pm = (\epsilon^2 - q^2 \pm i\alpha)/\lambda^2$ and $\lambda = (2i\alpha|\epsilon/q|)^{1/2}$.

Turning now to the applications of Eq.(5), we first discuss the electron spectral function satisfying the dispersion relation $G^R(\epsilon, \mathbf{p}) = \int A(\epsilon', \mathbf{p})d\epsilon'/\pi(\epsilon - \epsilon' + i\delta)$. Near the maximum, $|\epsilon^2 - \mathbf{p}^2| \lesssim \alpha$, it can be shown to take the form

$$A(\epsilon, \mathbf{p}) \approx \frac{(\epsilon\hat\gamma_0 - \mathbf{p}\hat\gamma)\sqrt{\beta}}{[(\epsilon^2 - p^2)^2 + \beta^2(1 + (|\epsilon/p| - 1)^4/4)]^{3/4}} \quad (13)$$

where $\beta \approx \pi\alpha/8$, thereby demonstrating a replacement of the bare pole by a branch cut of the function $(z - z_0)^{3/2}$ resulting from the above convolution procedure. According to Eq.(13), the decay of the electron propagator in real space ($G^R(\epsilon, \mathbf{r}) \propto \epsilon(\Gamma/r)^{1/2}e^{-\Gamma r}$ for $r \gg 1/\Gamma$) is governed by $\Gamma(\epsilon) \propto \alpha/\epsilon$ which should be thought of as the actual (energy-dependent) quasiparticle width.

This direct experimenta prediction can be tested by performing ARPES measurements in the VLL phase of the cuprates under the weak-field conditions ($\sqrt{H} \ll T \ll \Delta$).

Also, comparing Eq.(13) with the estimate for $\mathrm{Im}\Sigma_v$ obtained from Eq.(4), we conclude that in the ballistic regime both the BA and the Doppler scattering mechanisms appear to be equally important, contrary to the conclusions drawn in Ref. [16].

Next, we compute thermal conductivity given by the averaged product of two electron propagators

$$\kappa_{xx} = \int_0^\infty \frac{(\epsilon/T)^2 d\epsilon}{\cosh^2(\epsilon/2T)} \int \frac{d\mathbf{r}}{2\pi}\mathrm{Tr}\langle\hat\gamma_1\mathcal{G}^A(\epsilon, \mathbf{r})\hat\gamma_1\mathcal{G}^R(\epsilon, -\mathbf{r})\rangle. \qquad (14)$$

The corresponding path integral reads as

$$\langle\hat\gamma\mathcal{G}^A(\epsilon, \mathbf{r})\hat\gamma\mathcal{G}^R(\epsilon, -\mathbf{r})\rangle = \int_0^\infty d\tau_1 d\tau_2 \int_{\mathbf{r}_{1,2}(0)=0}^{\mathbf{r}_{1,2}(\tau_{1,2})=\pm\mathbf{r}}$$

$$\prod_{\alpha,\beta=1,2} D\mathbf{r}_\alpha D\mathbf{p}_\beta \hat\gamma e^{i\hat S_0[\tau_1]}\hat\gamma e^{i\hat S_0[\tau_2]} \prod_{i=a,v} W_i(\mathbf{r}_\alpha - \mathbf{r}_\beta), \quad (15)$$

where the factors $W_i(\mathbf{r}_\alpha - \mathbf{r}_\beta)$ with $\alpha \neq \beta$ account for the vertex corrections, alongside the self-energy ones ($\alpha = \beta$). Thus, the path-integral method of computing the Dirac fermion conductivity is capable of proceeding beyond the conventional (non-crossing and fan-shaped) ladder series of the vertex corrections to the bare fermion bubble (the latter suffice only if the number N of the Dirac species is large, while in the d-wave problem $N = \sum_\sigma 1 = 2$).

Upon integrating over the "center of mass" variables $\mathbf{r}_1 + \mathbf{r}_2$ and $\mathbf{p}_1 + \mathbf{p}_2$ and rescaling the ones describing relative motion, we again arrive at Eq.(11). This time around, it is formulated in terms of the transverse relative coordinate $(r_{1\perp} - r_{2\perp})$, and its solution yields the average $\langle\mathcal{G}^A\mathcal{G}^R\rangle \propto \epsilon(\Gamma_{tr}/r)^{1/2}e^{-\Gamma_{tr}r}$, where $\Gamma_{tr} = 2\Gamma$ is now playing the role of the momentum relaxation rate, in agreement with the above estimate for the quasiparticle spectral width. Plugging this asymptote into Eq.(14) we find that in the low-field regime the thermal conductivity behaves as

$$\kappa_{xx}(T, H) \simeq \frac{7\pi^2}{15}\frac{T^3}{H}. \qquad (16)$$

While being in agreement with the estimate based on the kinetic equation $\kappa \propto T\nu/\Gamma_{tr}$ proportional to the linear density of states (DOS) $\nu \propto T$ and $\Gamma_{tr} \propto H/T$, Eq.(16) is strikingly different from the result ($\kappa \propto T^2/H^{1/2}$) that one would obtain by naively assuming that the quasiparticle width remains constant ($\Gamma' \propto H^{1/2}$) up to the energies $\epsilon \sim T$ (cf. with Refs. [11]).

The analysis of the data of Ref. [17] taken in $YBa_2Cu_3O_{6.99}$ shows that Eq.(16) should be expected to hold for $T < 30K$ and $0.1 \lesssim \sqrt{H}/T \lesssim 1$ where the vortex-induced Γ_{tr} dominates over the other mechanisms of scattering, including potential impurities.

Our approach also enables one to compute other averages, such as $\langle\mathcal{G}^R\mathcal{G}^R\rangle \propto \epsilon(\Gamma_{tr}/r)^{1/2}e^{2i\epsilon r - \Gamma_{tr}r}$, which controls the effect of vortex disorder on superfluid stiffness measured by μSR

$$\rho_s(0) - \rho_s(T) = \frac{1}{\pi k_F}\mathrm{Im}\int d\epsilon \tanh\left(\frac{\epsilon}{2T}\right) \qquad (17)$$

$$\int d\mathbf{r}\mathrm{Tr}\langle\mathcal{G}^R(\epsilon, \mathbf{r})\mathcal{G}^R(\epsilon, -\mathbf{r})\rangle \simeq \frac{2\ln 2}{\pi}T + \frac{H}{8\pi T}\ln\frac{T^2}{H},$$

Notably, in contrast to the spectral and transport characteristics whose behavior is determined by the structure of the electron spectral function near its maximum, Eq.(17) is governed by the overall momentum integral of the solution of the two-particle analog of Eq.(11).

This behavior is common amongst the thermodynamic quantities associated with the averages of local bi-linear operators $\overline{\psi}_\sigma(\mathbf{r})\psi_{\sigma'}(\mathbf{r})$ which are invariant under the gauge transformation (1). In fact, such averages as, e.g., $G^R(\epsilon,\mathbf{0})$ must be computed differently, since now the trajectories contributing to the path integral (6) may deviate very strongly from the semiclassical one, $\mathbf{r}_0(\tau)=\mathbf{0}$, although the mean square of the distance over which a typical trajectory $\mathbf{r}(\tau)$ ventures from the origin still scales quadratically with time, $\langle\mathbf{r}^2(\tau)\rangle\propto\tau^2$. For $1/\Delta\lesssim\tau$ this allows one to evaluate the damping factors as

$$W_i[\mathbf{r}(\tau)]\approx\exp\left[-\frac{\tau^2}{2}\int\frac{d\mathbf{q}}{(2\pi)^2}w_i(\mathbf{q})\right]. \qquad (18)$$

Plugging (18) instead of (8) into (6) and computing the resulting (quadratic) path-integral, we arrive at the correction to the linear DOS corresponding to the clean limit

$$\nu(\epsilon)=\mathrm{ImTr}\left[\hat{\gamma}_0 G^R(\epsilon,\mathbf{0})\right]=\frac{\sigma_H}{\pi^{3/2}}\mathcal{F}\left(\frac{\epsilon}{\sigma_H}\right), \qquad (19)$$

where $\sigma_H^2\propto H\ln(\Delta^2/H)$ and $\mathcal{F}(x)=\pi^{1/2}x\,\mathrm{Erf}(x)+\exp(-x^2)$. The effect of vortex disorder is most pronounced at small energies $\epsilon\lesssim H^{1/2}$, and it appears to be stronger than in the semiclassical (Doppler-only) approximation (cf. with [18]).

Directly, this DOS correction can be extracted from the tunneling conductance $G(V)\propto\nu(V)$. Indirectly, it can also be manifested through the correction to electronic specific heat

$$C(T)=\int_0^\infty\frac{(\epsilon/T)^2\nu(\epsilon)d\epsilon}{\cosh^2(\epsilon/2T)}\simeq\frac{18\zeta(3)T^2}{\pi}+\frac{\sigma_H^4}{16\pi T^2} \qquad (20)$$

Notably, the correction $\Delta C\propto H^2\ln(\Delta^2/H)/T^2$ is smaller than the result ($\Delta C\propto H$) obtained under in the situation where the inter-vortex repulsion is stronger than random pinning, and therefore the VLL is partially ordered [18].

Although the smallness of the disorder-induced term in Eq.(20) might hinder its detection, an alternate possibility is offered by the spin-lattice relaxation time

$$\frac{1}{T_1(T)}\propto\int_0^\infty\frac{\nu^2(\epsilon)d\epsilon}{\cosh^2(\epsilon/2T)}\simeq\frac{2}{3}T^3+\frac{\sigma_H^3}{3\pi^{5/2}}, \qquad (21)$$

where we dropped the overall prefactor proportional to the ion-specific matrix elements.

In summary, we carried out a fully quantum mechnical analysis of the quasiparticle properties of the VLL phase of layered d-wave superconductors. We demonstrated that both the semiclassical Doppler shift and the intrinsically quantum mechanical BA scattering have comparable effects on all the observables. Our path-integral approach enabled us to identify the energy-dependent effective quasiparticle width $\Gamma(\epsilon)\propto H/\epsilon$ describing the near-maximum ($\epsilon^2\approx\mathbf{p}^2$) behavior of the distinctly non-Lorentzian electron spectral function which can be directly measured by ARPES.

Also, we exposed the striking difference between the two rates: $\Gamma_{tr}\sim\Gamma(T)\propto H/T$ displayed by the transport characteristics and $\Gamma'\sim\Gamma(H^{1/2})\propto H^{1/2}$ manifested by tunneling and thermodynamic quantities.

In contrast, the real-space averaging procedure applied in the previous studies of the disordered vortex states [18] is bound to deliver the latter rate, because, focusing solely on the energy distribution, it does not faithfully represent the momentum dispersion of the averaged electron spectral function.

We emphasize that the origin (that is, a non-trivial energy dependence) of the different apparent quasiparticle rates, as revealed by the different measurements, must be distinguished from the conventional juxtaposition of quasiparticle lifetime versus transport time in, e.g., photoemission and transport experiments.

We expect that, albeit derived under a number of simplifying assumptions, our main conclusions are robust against including such factors as spatial anisotropy and even non-linearity of the fermion dispersion as well as inter-node scattering and, most importantly, modifying the distributions (3) with the purpose of describing partially ordered "vortex glass" states [18].

Lastly, the results of this paper can also be used to describe the effects of thermal phase fluctuations controlled by $\alpha\propto T$ in the pseudogap phase of the cuprates [13,14]. To this end, we predict that different measurements may return different values of the effective quasiparticle width, which might explain the inconsistency between the widths deduced from the tunneling and ARPES data [13].

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