The Changing Relationships Between Science and Mathematics: From Being Queen of Sciences to Servant of Sciences

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It is fair to say that nothing epitomizes our modern life better than computers. For better or worse, computers have infiltrated every aspect of our society. Today computers do much more than simply compute; they have changed the way we conduct research and the way we have learned mathematical or scientific knowledge. Thanks to computers, mathematics has moved from its original position as the queen of sciences to being the servant of sciences. To fully understand and appreciate the impact of computers on our lives and the promises they hold for the future, it is important to understand and to compare the historical relationship between science and mathematics.

In this paper, we will give a brief historical overview of the relationship between science and mathematics then we will move on to the impact of computers in this relationship and finally we will talk about the future of this relationship. The origin of the sciences is rooted in tool making and agriculture. It is fair to say that making and using tools and the cultural transmission of scientific knowledge became essential to the existence of the human species and was practiced in all human societies. The history of science and mathematics starts with the Neolithic era. In the Neolithic Revolution, although mathematical knowledge was limited to counting and arithmetical operations, scientific knowledge was more advanced than mathematical knowledge; for example, potters possessed practical knowledge of the behavior of clay and fire, and, although they may not have had explanations for the phenomena of their crafts, they toiled without any systematic science of materials or self-conscious application of the theory to practice (James, & Thorpe, 1994). Of course, practical knowledge embodied in the crafts is different from knowledge deriving from some abstract understanding of a phenomenon. To change a light bulb, one needs direct instruction or hands-on-experience, not any special knowledge of electricity or energy. It seems fair to say that Paleolithic people applied practical skills rather than any theoretical or scientific knowledge to practice their crafts (Basalla, 1988; De Camp, 1963; & Usher, 1988).

After the Neolithic era, urban civilizations started to flourish in different parts of the world. During this time period, many civilizations emphasized the importance of hydrology and ecology, and they recognized the importance of intensified agriculture, abetted by large-scale hydraulic engineering projects, because water management was necessary to support the existence of civilization (De Camp, 1963; & Usher, 1988). Based on managing the annual flooding of the Nile, Egypt manifested all the earmarks of high civilization, including large-scale buildings, writing, mathematics, elementary astronomy, and expanded crafts. At the same time, the people of the Indus River Valley farmed the arid plains, and they built embankments to protect cities against erratic silt floods. In China, rice cultivation spread with the help of hydraulic control constructions. Also at this time period, the Mayan civilization also developed sophisticated artificial irrigation structures (James, & Thorpe, 1994).

In all these civilizations, mathematical methods developed along with writing out of practical needs. In Egypt, the origin of geometry stemmed from the need to resurvey fields after the flooding of the Nile. Although pure mathematics later became an abstract game played by mathematicians, the practical, economic, and craft roots of early mathematics remain visible in these applications. They used tables of exponential functions to calculate compound interest, and quadratic equations were solved in connection with other problems. Linear equations were solved to determine shares of inheritance and the division of fields. Lists of coefficients for building materials may have been used for quick calculations of carrying loads (Kirby, Withington, Darling, & Kilgour, 1990). And the calculation of volumes reflected no idle interest in geometry but was applied in the construction of canals and other components of the infrastructure. The only reason that they studied astronomy was the utility and necessity of accurate calendars in agrarian societies not only for agricultural purposes, but also for regulating ritual activities (James, & Thorpe, 1994). Their investigations were based on observation, mathematical analysis, and modeling of the phenomena, and no attention was paid to the abstract models of mathematical cycles. As opposed to the Greeks, they used knowledge for practical purposes, without a distinctive ideology that stressed the philosophical dimension of knowledge and a detachment from any social or economic objectives (Basalla, 1988).

The Ancient Greek civilization elevated mathematics to the level of the abstract and the theoretical, and they discovered the role of mathematics in proving their theories. Greek mathematics moved far away from practical arithmetic to pure arithmetic and geometry, by developing proof as a means and model for justifying claims to knowledge. Greek science tended toward abstract thought and many scientific fields came to life during Greek civilization. For the Greeks, mechanics itself was a subject to scientific analysis, and they articulated a theoretical and applied mathematical science. Archimedes, the ancient Greek genius, mastered the principles of simple mechanics such as the lever, wedge, screw, pulley, and windlass, and produced an analysis of balance including hydrostatic balance (Basalla, 1988; & De Camp, 1963).

Although many scientific and engineering applications did appear in different parts of the world after the Greek civilization, they lacked the natural philosophy characteristic of the earlier Greek civilization. Military engineering, hydraulics, and astronomical problems were the main stimulating factors that shaped scientific discoveries and mathematical knowledge (Pacey, 2001). Although Chinese and Islamic scientists used arithmetic and algebraic techniques, including simultaneous equations and square and cube roots to solve problems related to measurement of agricultural fields, and construction and distribution problems, they never developed a formal geometry, logical proofs, and they consistently displayed a practical trend in their emphasis on arithmetic and algebra. While they solved higher-order equations, many problems had roots in the practical world dealing with taxes, charity, and the division of inheritances. The construction of large dams, waterwheels, and irrigation canals all formed part of the Islamic and Chinese engineering repertoire (De Camp, 1963).

As this brief historical sketch has shown us, early mathematics generally were used for practical proposes and scientific knowledge was limited to application in agriculture, medicine, astronomy, physics, chemistry, civil engineering, and mechanical engineering. Engineering and scientific knowledge were still embodied in crafts (Kirby, Withington, Darling, & Kilgour, 1990). The adaptation of the present number system and the invention of printing made great contributions to scientific and mathematical knowledge. In the fifteenth century, new mathematical and scientific knowledge started to flourish in Europe due to urbanization (Kline, 1972). Sailors were in need of more accurate navigational techniques. This particular need sparked the development of trigonometry and non-Euclidean geometry for an accurate representation of the spherical earth's surface. Partly as a result of making navigational calculations and partly as a result of economical development, the need for complex calculations resulted in the development of logarithms by Nipper and the others. During this time period a gambling dispute caused Fermat to develop the foundation of probability theory. Nevertheless, none of these incidents made such an impact as did the development of calculus. The irony is that calculus was developed by two non-mathematicians –(Newton was a physicist and Leibniz was a philosopher) to solve physics, astronomy, and real world problems. Newton's inspiration to invent calculus was the need for a mathematical vehicle for discoveries in physics and astronomy (Eugene, 1960).

The development of calculus made a huge impact on scientific knowledge; expressing the rules of physics with mathematical formulations provided a key source of power for the Industrial Revolution and gave enormous impetus to the development of machinery of all types. Especially, the Military Revolution introduced competition between countries and a dynamic social mechanism that favored technical development. As a result, a new major classification of engineering dealing with tools and machines, namely mechanical engineering emerged. Eighteenth-century engineers benefited from scientific theory; technical developments provoked the interests of scientists and led to theoretical advances (Berlinski, 1995; & Boyer, 1949). However, the gulf between practical applications and theoretical research remained to be bridged until the beginning of the twentieth century (McClellan, & Dorn, 1999). In the same vein, although the fact that amber when rubbed will attract light objects was known by the Greeks, progress was made in the understanding and use of electrical energy as a result of the development of infinite series, differential equations, and complex numbers. In electrical engineering, $e^{jwt} = \cos wt + i \sin wt$ is equivalent in importance to the discovery of the circulation system of the human body or of the development of letters in writing. Although this equation was first produced by Euler, the development of electrical engineering made an impact on the development of mathematics. Marconi's understanding of waves helped him to invent radio; as a result, solving partial differential equations gained more importance (Kirby, Withington, Darling, & Kilgour, 1990). Later, Maxell mathematicized Faraday's ideas and gave the world the elegant mathematical expressions that describe the electromagnetic field in the form of wave equations, known as Maxwell's equations.

This historical interplay between mathematics and engineering is also true for civil engineering. Civil engineering developed into a scientific field along with the scientific revolution: the use of iron in buildings and iron bridges, the development of cast iron in textile mills, and the improvement of cement quality. At first, the use of long span roofs, suspension bridges, the truss design and high buildings forced engineers to solve large linear systems. And later development in soil mechanics, foundation engineering, hydraulic structures, mechanics, and dynamics stimulated the work toward finding solutions to differential equations (Pacey, 2001). Thus, it would be fair to say that engineering contributions helped mathematical developments in differential equations and numerical analysis. Historically, a significant number of mathematicians in the sixteenth and nineteenth centuries were also engineers such as Archimedes, Euler, the Bernoulli family, and Pioncare.

Around the late nineteenth century, engineering fields started to develop into scientific fields with the contribution of mathematics. The rapid rise of engineering science (both static and dynamical) in the nineteenth century extensively altered the practice of engineering and lent considerable impetus to the evolution of mathematics. Before this time period, Cauchy gave calculus a logically acceptable form by setting it on a more rigorous basis. Also, mathematicians became pretty comfortable in using complex numbers. The interactions that lead to these developments are like a conversation in which incomplete information sparks new ideas and what we can call responsive inventions. Due to pressure coming from science, mathematics started to develop into different research paradigms: algebra, topology, numerical analysis, number theory, geometry (algebraic, differential, and analytic geometry), ordinary and partial differential equations, and probability (Davis, & Hersh, 1981). Among these topics, numerical analysis and differential equations gained importance because of the need to solve them in engineering problems. Also, the technological race during the two World Wars was a major stimulating factor for further development of mathematics. Finally, the developments in numerical analysis techniques and the need for solving ordinary and partial equations encouraged the governments to support the development of computers. In the United States, the government supported the development of computers during World War II, because highly complex computations were required to develop the atomic bomb. During this period, due to security concerns in military communications, number theory gained importance (as the basis for cryptography) and it moved from being the game of mathematics to being a research field in mathematics. Probability also gained importance during this time. The Germans used and developed probability theory to decide where they were going to locate their antiaircraft guns. Before this application the development of statistical analysis was the main stimulating factor for probability theory.

Although historically, science and engineering fields contributed major developments in mathematical theories, today they owe their existence to mathematics. The situation has changed dramatically in the past decade or so. For these fields, mathematics is more than another tool that they can use to solve their problems or give an account for their scientific discoveries (Boyer, 1949). Mathematics is now a key component of these academic fields and it is almost impossible to do research in these fields without using mathematics. It will be fair to say that computers and mathematics together are the hidden heroes behind the huge amount of recent scientific developments and discoveries. It is impossible to conceive of present-day technological achievement without the previous invention and availability of infinitesimal calculus and its role in celestial mechanics, astronomy, and engineering. It is well known that the development and the use of mathematical tools were a necessary prerequisite and stimulus to today's technological achievement. In addition to assisting the development of machines, construction of bridges, and design of electric motors, the techniques of calculus helped to formulate the theories of thermodynamics, electric fields, and construction of satellites (McClellan, & Dorn, 1999).

Today, having mathematical knowledge of advanced calculus is required in almost all scientific fields and engineering majors. Today, even biologists are in need of using very sophisticated mathematics in their new areas (*e.g.* molecular biology, epidemiology and immunology) to do their investigations. Medicine also requires the use of mathematics such as in physiologists' modeling of solute and water transport by using renal functions. Biochemists use mathematics for enzyme kinetics, solving the Michaelis-Menton equation. On the other hand, microbiology uses mathematics in calculations for growth media, estimation of cell growth and biomass, modeling batch cultures and continuous growth, and control of microbial growth. Chemical Engineers use mathematics in their calculations of material and energy balances, transpiration of phenomena and kinetics, through formulation and solution of ordinary and partial differential equations. Analyzing the dynamic behavior of physical systems also requires modeling and solving differential equations. Today, collaboration among theoretical and applied mathematics and science is essential for scientific progress. Experimental research is linked to mathematical modeling so that observations about the real world can be interpreted and new hypotheses for testing can be generated.

Business also uses a great deal of mathematics. Mathematical theories are being used in the calculation of stock prices within seconds, thus offering the investor the possibility to implement hedging strategies with almost instantaneous adjustments. In addition, more sophisticated financial products have required deeper theories, which are based on mathematical modeling to price them. Gerald Debreu, a mathematician, developed the theory of equilibria (which is fundamental for the theory of missing arbitrage) and he was awarded the Nobel Prize for economical sciences in 1983. Later, Fischer Black and Myron Scholes realized the importance of stochastic calculus for describing stock markets and developed one of the most important theories for options pricing: the famous Black-Scholes model.

The impact of mathematics on science, business, and engineering can be summarized by the following comments: use of mathematical terms to express the ideas of science and engineering to prevent ambiguity, expressing the findings in nature and engineering mathematically to verify or disprove experimental results, and expressing scientific and engineering ideas with very concise statements using the symbolism of mathematics. Once an idea is expressed in mathematical form, we can use the axioms, the definitions, and the theorems of mathematics to change it into other statements. In some way, mathematics mechanizes our thinking and then the computer makes it possible to process information almost instantly.

Computer sciences also use mathematical theories. The designing and computing operations of electronic computers themselves involve ideas of mathematical logic and combinatorial analysis. The invention of the computer, more than any other single achievement, marks the change in the relationship between mathematics and science from that of queen to servant. The relationship between computer science and mathematics is symbiotic. Thus a chain of development of this technological tool may be traced back though some of the major figures of early modern mathematics, science and technology (Pacey, 2001). Computer science owes its existence to mathematics. Leibniz probably never foresaw how his invention of the binary system would effect the creation of the computer era. The real beginning of the computer era starts with Charles Babbage, a mathematician, who noticed a natural harmony between machines and mathematics and realized that machines were best at performing tasks repeatedly without mistake (Atiyah, 1986). Today's computer science would not exist without the contribution of the Boolean algebra system and the contributions of the famous mathematicians von Neumann and Boole. Mathematics provides the theoretical foundation for computer science. So, it is not surprising that mathematics finds its way into computer science curricula, at both the undergraduate and graduate levels. Computer science curricula are heavily dependent on counting techniques, number theory, logic and proofs, and mathematical induction.

The relationship between computer science and mathematics is obvious in number theory and numerical analysis. They are simultaneously branches of applied mathematics and branches of computer science, which is the art of obtaining numerical answers to certain mathematical problems. Although, the origin of numerical analysis goes back to the Babylonians in their simple numerical techniques to approximate the square root of 2, it gained its importance as a mathematical research field after World War II. The origin of number theory goes back to the ancient Greeks, and it also gained importance as a mathematical research field after World War II. World War II provoked a vast amount of computing that stimulated the development of many new techniques. The computer was born after the war ended and lent tremendous impetus to numerical analysis. As computers developed, they made a huge impact on mathematics (White, 1978). At first, different numerical methods developed to solve nonlinear systems such as Newton's method, the Quasi-Newton method, steepest descent method, and homotopy and continuation method. The basic reason for developing different methods was to find a better and faster way to solve nonlinear systems. Since early computers were relatively slow in their processing and saving space in memory was important, mathematicians were stimulated to find more efficient methods. With the technological capabilities of today’s computers, these issues have lost their importance. The same argument is also true for mathematical developments in iterative techniques in matrix algebra. We can now invert a (million by million) matrix, solve large systems of simultaneous differential equations, solve boundary-value problems of partial differential equations with powerful computers (Graham, Patashnik, & Knuth, 1994). On the other hand, although number theory owes its development to computers, it is making a huge contribution to computer technology by supplying necessary tools and theories to decrease communication time between computers and to overcome security concerns when using computers (especially in an Internet environment) through data encryption techniques. The computer has made a huge impact on doing and learning mathematics. It helped mathematicians prove famous unsolved problems, such as the proof of the four-color problem in topology. The effective use of instructional software helps students learn mathematics in a meaningful way.

The invention of computers completely revolutionized the relationship between mathematics and the other sciences (Mitcham, 1994). Although the mathematics that is taught at most engineering universities has not been changed for a very long time, with the shift toward greater use of numerical tools in many engineering subjects, the content of mathematics is undergoing profound changes, brought about by an emphasis on mathematical modeling. Today's engineers and scientists are heavily involved in the development and use of new materials and technologies, especially in computer-aided engineering. The computer-based simulations bring a new and useful tool to science and engineering. New system configurations and products can be designed and developed and in the later stage tested through computer simulations. Modeling in the engineering subjects and in engineering education has changed through the use and development of computers. The need for using computers and mathematics is obvious in scientific fields and in the engineering curriculum. The use of computers to solve equations of engineering problems has become routine in engineering practice. Simulation as an engineering tool has grown so rapidly because it is much cheaper than building prototypes and testing them (Graham, Patashnik, & Knuth, 1994). Mathematical modeling by using computers reduces a complex reality to a more simple method by identifying essential elements, linking them conceptually, and seeing how they interact. The invention of the integrated circuit and well developed theories in applied mathematics made it possible to bring computing power into different scientific fields (Bruijn, 1986). The power of using computer and mathematical models was demonstrated in their application in the guidance system of the Minuteman ballistic missile. The development of this guidance system also facilitated the human journey to the moon through the on-board guidance and control computer for the Apollo spacecraft.

Numerical simulation technology has advanced many areas including aerospace, chemical, communication, manufacturing, medicine, semi-conductor processing, and transportation. A numerical solution to scientific and engineering problems can also be obtained by using the finite element method (Kline, 1972; Atiyah, 1986). The finite element method provides a realistic simulation of engineering and scientific problems. Thanks to these technologies, we can emphasize problem-solving techniques and use realistic engineering examples to demonstrate the relevance and utility of mathematics to engineers and scientists. Computers are needed to gather relevant information, solve problems, and anticipate data requirements and present information visually.

The spread of the computer as a powerful new technology after 1980 altered the scientific, engineering, and mathematical landscape of the world (Borgman, 1984). The new medium created a communication revolution that increased the amount and accuracy of information available and made knowledge available to so many others. And, just as previous media (such as printing) did, this new medium has been remaking things since the time it came into society (Mitcham, 1994). The technology of the computer produced a huge impact on contemporary science with corresponding input from science and mathematics on computer technology. Today, even social scientists are becoming more dependent on mathematics and computers when they are unable to make precise measurements (Bruijn, 1986). High-speed computers are valuable tools in the development of mathematical models. They enable us to mechanize some of the process of scientific thinking itself. In statistics, mathematical theory tries to give a rationale for selecting a procedure for analyzing the data rather than relying on intuition.

The Internet will facilitate communication and cooperation among academics who may not otherwise be aware of each other's research. The ability of computers, to convey data, and make millions of complicated calculations in fractions of a second will be an empowering technology that will continue to produce dramatic social and cultural consequences for engineers, scientists, and mathematicians (White, 1978). The on-line distribution of scientific information will dissolve the traditional constraints of time and will solve the obstacles created by the paper journals' control of the information and its distribution. Most of the time, articles may be revised many times and publication of research findings takes more than one year. The Internet, producing knowledge at a lower cost and greater speed, contributes decisively to the diffusion of scientific and mathematical knowledge. As a result, the effect of new scientific and mathematical activities on the development of modern scholarship will be intensified.

This new medium will open our eyes to new possibilities and invites scholars to think freshly about the future of science and mathematics by challenging software developers to build products that better support the scientists' and mathematicians' needs (Mitcham, 1994). But computers, and specifically the Internet, do not simply influence our culture and society; they are themselves inherently cultural and social. If there is to be any reconciliation between science and mathematics, it will come from connecting them with mathematical and scientific innovations.

New technologies have been changing the classroom dynamics in the mathematics classroom by changing ways of communication and teaching, as well as by extending ways of learning. Since communication is necessary for successful mathematics education, the role of the Internet in mathematics education is becoming crucial (Owston, 1997). The Internet is being used in mathematics education as a resource for information, as a tool for mathematics learning, as a medium for classroom demonstration, and as a communication tool. The use of Java applets can provide effective problem-solving opportunities focused on each particular student's needs by running simulation experiments to illustrate mathematical concepts. They can be used as demonstrations in large lectures or, with some guidance, used by students to explore these concepts (Houston, 1998; Zhao, 1998).

Mathematics teachers can benefit from the Internet to create a professional community to provide the opportunity for reflection through dialogue with their colleagues (Novick, 1996; Schrum 1996). In the coming years, The Internet will continue to serve as a virtual library for mathematics education (Clark, et al., 1998; Noss, & Hoyles, 1996). Everyday, the number of web pages related to teaching and learning mathematics, mathematics research results, discussion groups, curriculum projects, on-line mathematics courses, and Java applets will continue to increase. Virtual classroom sites will flourish to allow students not only to explore and investigate mathematical ideas but also to contribute to students' active construction of knowledge (Fetterman, 1998). Dynamic Java applets, which allows students to explore a mathematics idea, will be more developed, allowing students the opportunity to investigate solving nonlinear systems or finding solutions to differential equations on web.

When considering computers as an added instrument to enhance engineering and science education and scientific development, a holistic view needs to be taken to ensure that the computers can be used for the purpose for which they were intended. Our conclusion is that mathematics has changed science and engineering much more than science and engineering has changed mathematics. Today's scientists and engineers rely on mathematics not merely as a tool for calculation but as a source of inspiration.

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