REDUCTION OF THE OUTPUT IMPEDANCE OF PWM INVERTERS FOR UNINTERRUPTIBLE POWER SUPPLIES

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ABSTRACT:

Inverters in UPS systems feeding nonlinear or step-changing load must be capable of providing the output voltage waveform quality in spite of all load and line disturbances. Switching algorithm applying output filter state feedback in order to control the output voltage instantaneous value is presented in this paper. Algorithm requires a synchronous pulse-width modulator to be placed in the feedback loop. Modulator introduces nonlinearity, and special attention is paid to stability limits of the resulting nonlinear discrete time system. Results are applicable on any converter controlled by the pulse width modulator in the closed loop.

INTRODUCTION

Static PWM voltage source inverters are frequently encountered in uninterruptible power supply systems. Critical equipment such as computers, communication systems and medical instrumentation require high quality power supplies. Inverter output voltage should be near-sinusoidal and low order harmonics attenuation must be sufficient to suppress the influence of the load current distortions on the output voltage. The development of high switching rate power switches enabled the construction of inverters with light output L-C filter and the implementation of various switching algorithms. Regular-sampled PWM technique [2] gives well defined output voltage spectrum and superior static characteristics for linear loads, but lacks the tolerance to nonlinear loads, such as rectifier bridges. As in all feed-forward programmed approaches, step load changes may cause large overshoot voltage transients while line and load disturbances directly reflect on the output voltage quality. Sensitivity to load disturbance is defined by the inverters output impedance at relevant frequencies. Sliding-mode switching algorithms [3],[4] applying output filter state feedback in order to control the output voltage instantaneous value give excellent disturbance rejection and minimize inverter output impedance , but bring inherent randomness in commutation. The randomness gives a rise to beat frequencies and sub-harmonics in the output voltage spectrum.

Output impedance may be reduced and the quality of the output voltage spectrum preserved if the inverter switches are controlled by the switching algorithm presented in this paper. Regular-sampled pulse width modulator is placed in the feedback loop controlling the output voltage instantaneous value. Modulation samples consist of regular sinusoidal signal and sampled output filter states. The paper presents theoretical background, design considerations and experimental results. Attention is paid to the selection of the feedback coefficients that minimize inverter output impedance and ensures stability in large. Proposed control structure is experimentally proved on the 1kW laboratory prototype of the transistorized voltage source inverter with L-C output filter. Inverters output impedance is measured by injecting the load currents at relevant frequencies into the output terminals for open- and closed-loop case.

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2. DESCRIPTION AND ANALYSIS OF THE PROPOSED SWITCHING ALGORITHM

Inverter bridge, output L-C filter and pulse-width modulator form a closed-loop system controlling the output voltage (Fig. 1.). Sampled data of the referent waveform and the output filter variables serves as the modulation signal *Um(kT)* that linearly affects the width of the pulses appearing across the inverter bridge (Equ.(1)) in time intervals determined by the sampling frequency.

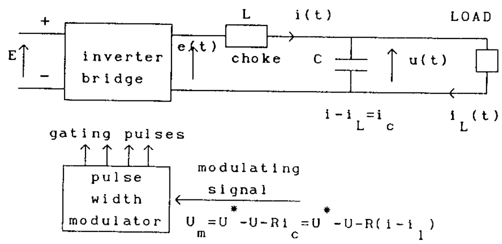

Fig. 1. : PW Modulator with filter states feedback

Setting the sample time T so that the sampling rate is commensurable with the output frequency, the quality of the spectrum is inherently obtained. The modulator sets the pulse polarity according to the sign of the modulating signal samples $Um(kT)$ and the pulse width 'τ_k' according to the magnitude:

$$\tau_k = \begin{cases} T & ; \ |U_m(kT)| \geq \alpha \\ T\dfrac{|U_m(kT)|}{\alpha} & ; \ |U_m(kT)| < \alpha \end{cases} \tag{1}$$

$$U_m(kT) = U^*(kT) - U_c(kT) - R\, i_c(kT) \tag{2}$$

The ratio $G=E/\alpha$ plays the role of the feedback gain, while parameter R determines the amount of the capacitor current feedback ($Ri_c=Ri_{ind}-Ri_{load}$). The matrix differential equation (3), describing the system in the continuous time, may be translated into difference form (4):

$$\frac{d\vec{x}}{dt} = A\vec{x} + B\,e + N\,i_{L\,(load)}\ ; \tag{3}$$

where:

$$\vec{x} = [u, i]^T \ ; \quad B = [0, \tfrac{1}{L}]^T \ ; \quad A = \begin{bmatrix} 0 & 1/L \\ -1/C & 0 \end{bmatrix} ;$$

$$N = [-\tfrac{1}{C}, 0]^T \ ;$$

Fundamental matrix of the system is $\phi(t)=\exp(At)$. Transition from $t=kT$ to $t=(k+1)T$ is given by the equation (4):

$$\vec{x}_{k+1} = \phi(T)\,\vec{x}_k + E\,sgn(U_{mk})\,\phi(T)\int_0^{\tau_k}\phi(-t)\,B\,dt + \phi(T)\int_0^{\tau_k}\phi(-t)\,N\,i_L(t)\,dt \tag{4}$$

$$\phi(T)\int_0^{\tau_k}\phi(-t)B\,dt = \begin{bmatrix} 2sin(\omega\tau_k/2)\ sin(\omega(2T-\tau_k)/2) \\ 2sin(\omega\tau_k/2)\ cos(\omega(2T-\tau_k)/2)\ \frac{1}{Z} \end{bmatrix} \tag{5}$$

$$\phi(T)\int_0^{T}\phi(-t)N\,dt = \begin{bmatrix} -Zsin(\omega T) \\ 1-cos(\omega T) \end{bmatrix} ; \quad Z=\sqrt{\frac{L}{C}}\ ; \quad \omega=\frac{1}{\sqrt{LC}}\ ; \tag{6}$$

Equation (4) is nonlinear because the elements of the transfer matrices are not constants. They are dependent on the pulse width τ_k and, hence, modulation depth $m=Um/\alpha$. Transfer functions in z domain could be derived by linearization, while the stability analysis requires application of the Liapunov's second method [5], [6], [7]. Linearization can be done if we assume that system states and modulation depth exhibit small excursions (Δu, Δi, Δm) around their stationary values (u_o, i_o, m_o). Equation (7) represents linearized equation (4):

$$\begin{bmatrix} \Delta u_{k+1} \\ \Delta i_{k+1} \end{bmatrix} = \begin{bmatrix} cos(\omega T) & Z\,sin(\omega T) \\ -sin(\omega T)/Z & cos(\omega T) \end{bmatrix} \begin{bmatrix} \Delta\,u_k \\ \Delta\,i_k \end{bmatrix} + \begin{bmatrix} -Z\,sin(\omega T) \\ 1-\,cos(\omega T) \end{bmatrix} \cdot$$

$$\cdot\ \frac{i_{L_k} + i_{L_{k+1}}}{2} + \frac{E}{\alpha}\,\omega T \begin{bmatrix} sin((1-m_o)\omega T) \\ cos((1-m_o)\omega T)/Z \end{bmatrix} (-\Delta u - R\Delta i + Ri_{L_k}) \tag{7}$$

Output impedance $Imp(z)=u(z)/i_L(z)$ and characteristic polynomial $f(z)$ are given by expressions (8) and (9) for the case $m_o=0$:

$$f(z) = z^2 + z\,(G\omega T\,sin(\omega T) + G\omega T \frac{R}{Z} cos(\omega T) - 2\,cos(\omega T)) + 1 - \frac{R}{Z} G\omega T \quad (8)$$

$$Imp(z) = \frac{a(z)(z - cos(\omega T) + G\omega T \frac{R}{Z} cos(\omega T)) + b(z)}{f(z)} \quad (9a)$$

where:

$$a(z) = GR\omega T sin(\omega T) - \frac{z+1}{2} Z sin(\omega T);\; b(z) = (G\omega T \frac{R}{Z} cos(\omega T) + \frac{z+1}{2}(1 - cos(\omega T)))\,(Z sin(\omega T) - RG\omega T sin(\omega T))$$

At the frequency f, output impedance $Z_{out}(f)$ is:

$$Z_{out}(f) = |\, Imp(exp(\,j\,2\pi fT)\,|\quad (9b)$$

If we assume that the sampling rate is infinitely large, the output impedance is:

$$Z_{out}(j\omega) = \frac{1}{j C\omega + \frac{1 + G(1 + jRC\omega)}{jL\omega}} \quad (9c)$$

Output impedance and the roots of the polynomial $f(z)$ depend on the output filter parameters (ω, Z), sampling rate and feedback coefficients ($G=E/\alpha$ and R). Regulator design consist in selecting G and R so that the system is stable in large and that output impedance at relevant frequencies is minimized. It's clear from expression (9c) that impedance is lower for higher gain. Hence, feedback gain should be set to the highest possible value that still insures stability in large and gives response with sufficient damping.

3. STABILITY ANALYSIS

Linearized system (described by the equation (7)) is stable if the gain G is lower than G_{max}:

$$G_{max} = \frac{2\,(1 + cos(\omega T)\,)\,/(\omega T)}{\left[\frac{R}{Z} cos((1-m_o)\omega T) + sin((1-m_o)\omega T) - sin(m_o\omega T) + \frac{R}{Z} cos(m_o\omega T)\right]} \quad (10)$$

Expression (10) can serve as an estimate for accessible gain, while stability for large disturbances must be analyzed more rigorously. From equations (4),(5) and (6), nonlinear transfer matrix can be derived:

$$\vec{x}_{k+1} = [T]_{(m)}\;\vec{x}_k \quad (11)$$

where:

$$t11 = cos\,(\omega T) - \frac{G}{m}\,2\,sin\,(\omega mT/2)\,sin\,(\omega(2-m)T/2)$$

$$t12 = Zsin(\omega T) - \frac{G}{m}\,2R\,sin\,(\omega mT/2)\,sin\,(\omega(2-m)T/2)$$

$$t21 = -\frac{1}{Z}sin(\omega T) - \frac{G}{m}\,\frac{1}{Z}\,2\,sin(\omega mT/2)\,cos(\omega(2-m)T/2)$$

$$t22 = cos\,(\omega T) - \frac{G}{m}\,\frac{R}{Z}\,2\,sin(\omega mT/2)\,cos(\omega(2-m)T/2)$$

Taking a positive definite matrix P, stability in large is insured if the function $V_k = x_k^T P\,x_k$ has negative increments $\Delta V_k = V_{k+1} - V_k$:

$$\Delta V_k = x_{k+1}^T P x_{k+1} - x_k^T\,P\,x_k = -\,x_k^T\,(-T^T P\,T + P)\,x_k \quad (12)$$

This condition is satisfied when matrix $-T^T P\,T + P$ is a positive definite ($t11>0$ and $\det(T)>0$). Since the coefficients of the matrix T are functions of the modulation depth m, condition must be tested for all values $-1<m<1$ as well as for the case of over-modulation ($m>1$). Matrix P can be determined from the equation (13):

$$-\,T^T_{(m=0)}\,P\,T_{(m=0)} + P = I\ (identity\ matrix) \quad (13)$$

Maximum allowable gains for different modulation depths m, calculated from equation (10) and condition (12) are given in Table I :

Table I. : Max. accessible G for $\omega T=.1$, $Z=30\Omega$ $R=3\Omega$

m	0.	.1	.2	.3	.5	.7	1	2
G_{max} (equation 10)	133	142	153	166	199	249	400	...
G_{max} (Liapunov's method)	131	136	141	147	159	171	179	258

4. SELECTION OF THE FEEDBACK COEFFICIENTS

The output filter state feedback is introduced for the purpose of reduction of the inverters output impedance. Therefore, it's necessary to determine optimal setting of the feedback gains G and R. From the expression (9c), minimization of the output impedance requires maximum permissible gain G. To prove this, the output impedance at relevant frequencies is experimentally measured on the inverter prototype (the prototype is described in chapter 5). Instead of the load, series connection of the variable frequency voltage source inverter and the load resistor is connected at the output terminals of the prototype. Using variable frequency inverter as the simulation of the nonlinear load, rated current is injected at different frequencies and the influence on the inverter prototype output voltage is measured. Results (p.u. output impedance at specific frequency) are contained in Table II:

Table II: Output impedance versus gain G at harmonics from 2 to 9 and T=100μs, ωT=0.1, Z=30Ω, R=3Ω

f(Hz)	50	100	150	200	250	300	350	450
G=0	.237	.707	5.68	1.48	.731	.503	.391	.275
G=5	.036	.076	.125	.191	.295	.489	.933	.79
G=10	.019	.040	.063	.091	.123	.167	.228	.466
G=20	.010	.021	.031	.043	.057	.072	.089	.135
G=40	.005	.011	.016	.021	.027	.033	.040	.055
G=100	.002	.004	.006	.008	.010	.012	.015	.019

Output impedance in the open loop (G=0) at the third harmonic is as high as 5 p.u., due to the filter resonant phenomena. This means that third harmonic load current of 0.2 p.u. produces voltage distortion of 1 p.u.. It can be concluded from the Table II that the gain should be set as high as possible, since the increase of the gain reduces the output impedance at relevant frequencies. This conclusion is in agreement with equation (9c). Increase of the gain, on the other hand may produce low damping and oscillatory response, and eventually cause instability. Design procedure presented hereafter gives a guideline for the selection of the coefficients G and R.

For m_o=0, the eigenvalues of the linearized system are the roots (z_1 and z_2) of the equation (14):

$$z^2 + b\,z + c = 0 \tag{14a}$$

where: $$b = G\omega T(\sin(\omega T) + \frac{R}{Z}\cos(\omega T)) - 2\cos(\omega T) \tag{14b}$$

$$c = 1 - \frac{R}{Z} G\,\omega T \tag{14c}$$

The character of the output voltage response on load disturbances is completely determined by the poles z_1 and z_2. These poles must lie inside the unit circle of the z-plain. For well damped response, it's desirable that the poles are located in the right half-plane, close to the 0-1 segment of the real axes. Therefore, it is to be investigated what feedback gains can be reached under the imposed limitations. Since $z_1 z_2 = c$, the coefficient c (14c) must be positive or zero (otherwise, either z_1 or z_2 is real and negative). Also, since the real parts of the z_1 and z_2 and, hence, their sum must be positive, the coefficient $b = -(z_1+z_2)$ (14b) must be negative or zero (for $-1<z<0$, system response is oscillatory, and large inductance current overshoot could be expected). Hence, the feedback gains are limited by:

$$G \le \frac{2\cos(\omega T)}{\omega T(\sin(\omega T) + \frac{R}{Z}\cos(\omega T))} \tag{15a}$$

$$G \le \frac{Z}{R\,\omega\,T} \tag{15b}$$

Maximum gain for the positive real parts of the roots z_1 and z_2 is achieved for:

$$b = 0\ ; \quad c = 0\ ; \quad z_1 = z_2 = 0\ ;$$

$$R = Z\ \ tg\,(\omega T) \tag{16}$$

$$G = \frac{1}{\omega T\ tg\,(\omega T)} \tag{17}$$

Characteristic polynomial of the closed-loop system is $f(z) = z^2$, and both closed loop poles are zero ($z_1=z_2=0$). Hence, under the constrain that the real parts of the closed loop poles must be non-negative, maximum permissible gain and minimum of the inverters output impedance is achieved when the proposed output voltage regulator performs dead-beat control. For a given sampling time T (e.g. switching frequency of the inverter bridge) and defined parameters of the output L-C filter, selection of the feedback gain G and R is very simple (expressions (16) and (17)). It's necessary to know only the angle ωT and the filter characteristic impedance Z.

The proposed parameter setting is based on the analysis of the linearized system. Large disturbance analysis requires the system nonlinearity to be taken into account. Stability in large should be tested as proposed by expressions (11)-(13) and must be ensured for all operating conditions expected. According to results in Table I, stability is critical for zero output voltage. Hence, it's sufficient to check stability in large for $m_0=0$. For the inverter prototype under the test (described in part 5), for gains $G<131$ (Table I), stability in large is insured for all operating conditions. The optimal gain $G=100$ (according to (17)) stands far below G_{max} and insures stable response for all magnitudes of load and input disturbances.

5. EXPERIMENTAL RESULTS

Proposed switching algorithm is aimed for the control of the voltage source PWM inverters with L-C output filter. In order to evaluate static and dynamic characteristics of the proposed structure, non-linear load and load-step experiments are performed on the laboratory prototype. For the purpose of comparison, all the measurements are done for the case when the inverter is controlled by the proposed switching algorithm, and for the case when conventional open-loop PWM control is applied.

The prototype parameters are given in the following table:

DC voltage	**400 V**
output voltage	**220 V rms**
output frequency	**50 Hz**
sample time	**100 μs**
rated power	**1 kW**
rated current	**5 A**
choke L	**30 mH**
capacitor C	**33 μF**
feedback gains	$R=3\Omega$ $G=100$

The main objective of the closed-loop PWM control is reduction of the inverters output impedance at relevant frequencies. Therefore, the output impedance at the frequencies from 50 to 450 Hz (from the fundamental frequency to ninth harmonic) is measured for both open-loop and closed-loop operation. The measurement is done in the following manner: 1) The output voltage reference is set to zero. 2) The sinusoidal current of the appropriate frequency and the amplitude 1 p.u. is injected into the inverter output terminals. 3) The voltage deviation produced in such a way is measured. Since the current of the rated amplitude is injected, the output impedance at given frequency equals p.u. value of the measured voltage deviation. Reduction of the output impedance is illustrated by Table III:

Table III: Reduction of the output impedance by the output filter state feedback

frequency (Hz)	50	100	150	200	250	300	350	450
Z^{open}_{loop} (%)	24	71	568	148	73	50	39	28
Z^{closed}_{loop} (%)	.20	.40	.60	.80	1.0	1.2	1.5	1.9
$\frac{Z_{open\ loop}}{Z_{closed\ loop}}$	118	176	946	185	73	42	26	15

The resonant frequency of the laboratory prototype is $f_r \approx 160$Hz. Therefore, the open-loop impedance exhibits the resonant peak for the third harmonic (150Hz). The closed-loop impedance is not affected by the output filter resonance and slightly increases with frequency. The last row in Table III gives measured reduction of the output impedance produced by the output filter state-feedback.

The output voltage response on the load step change is tested for the case of pure resistive load. Load is turned on by the triac, when the output voltage is at the maximum. The triac ceases to conduct at the first zero crossing of the load current, when the output voltage is zero. Oscilloscope traces of the output voltage and the load current are given in Figs. 2 and 3 for both open-loop and closed-loop cases. It's clear that proposed switching algorithm reduces voltage transients and eliminates output filter resonant phenomena.

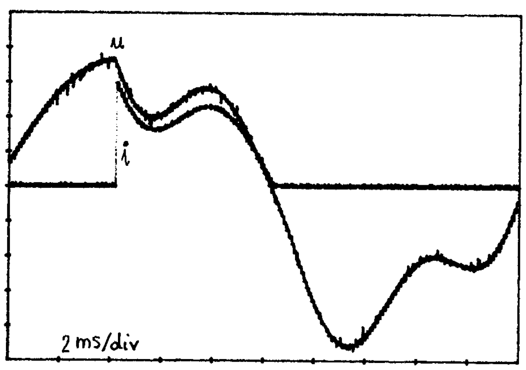

Fig. 2.: Load step response for the open-loop case (100 V/div, 2 A/div, 2 ms/div)

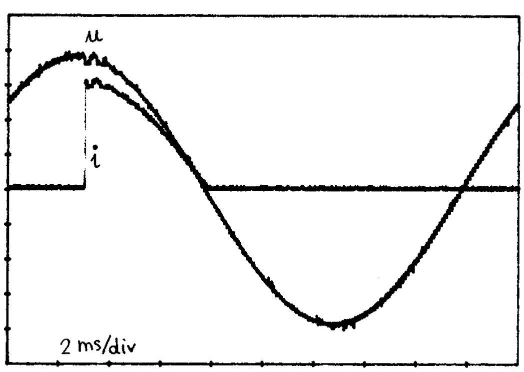

Fig. 2.: Load step response for the closed loop case (100 V/div, 2 A/div, 2 ms/div)

6. CONCLUSION

Voltage source inverters with output L-C filter, used in UPS systems are sensitive to current harmonics generated by the load. Switching algorithm for closed-loop control of the output voltage is proposed and design guidelines derived. Introduction of the proposed output filter state feedback dramatically reduces output impedance and eliminates overshoots that appear due to the filter resonance. The paper contains a detailed analysis of the pulse width modulator placed in a closed loop and gives straightforward procedure for the selection of feedback gains. Results of the analytical design are verified through the experimental investigation on the laboratory prototype. The same design guidelines could be applied with no changes to DC/DC converters containing pulse width modulator in the closed loop.

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