INSTANTANEOUS FEEDBACK IN VOLTAGE SOURCE INVERTERS : A COMPARATIVE STUDY BETWEEN NONLINEAR AND LINEAR APPROACH

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ABSTRACT

Voltage source inverters used in UPS systems feeding nonlinear loads must be capable of providing the output voltage waveform quality in spite of all line and load disturbances. Switching algorithms applying output filter state feedback in order to control the output voltage instantaneous value are discussed in this paper and the two approaches are compared. In the case of nonlinear controller, the driving force is switched at maximum amplitude into the output filter, while linear discrete-time controller encompasses a synchronous pulse-width modulator placed in the feedback loop. The former gives a higher tolerance level to load disturbances, but brings on randomness in commutations. Simulations and experiments performed show that the latter combines both excelent dynamic performance and synchronous switching. Theoretical background and design considerations are included for both solutions.

I INTRODUCTION

The development of high switching rate power transistors enabled the construction of light output filter VSI and the implementation of various control techniques. Feed-forward programmed waveform approach results in a well defined spectrum and superior static characteristics for linear loads, but lacks the tolerance to nonlinear loads, such as rectifier bridges, and suffers from large overshoot and undershoot voltage transients resulting from step load changes that cannot be tolerated by the newer computer systems. Output filter state feedback proposed in the paper remarkably reduces inverter output impedance, supresses voltage transients and eliminates filter resonant phenomena. Instantaneous feedback can be applied in a nonlinear or a linear manner (Fig. 1). The former, known as the "bang-bang" control and extensively described in the literature is extended here to the case of an arbitrary order output filter. This paper also presents the linear discrete-time approach. Nonlinear feedback controller contains a comparator with time-delay or hysteresis. Switches are controlled according to the sign of appropriately selected linear combination of the filter states and the output voltage error. A similar function serves as the input to the synchronous pulse-width modulator placed in the feedback loop in the case of linear approach. Basic concepts of both approaches are given in sections II and III, along with stability analysis and control circuitry design procedure. Experimental and simulation results are included in section IV while section V contains the conclusion of the present study.

I BASIC CONCEPT OF NONLINEAR APPROACH

The aim of an instantaneous feedback loop is to minimize the difference between the output voltage and the referent waveform (u^*) in the presence of load disturbances and parameter variations. The nonlinear controller fullfiles this task by switching the driving force at maximum amplitude. Since the driving force resembles the sgn function, Liapunov's direct method is easy to employ. The synthesis consists in choosing the argument of the sgn function as a linear combination of the error and the states. Closed loop transient response is completely determined by selecting the state feedback coefficients. A general approach for VSI with output filter entailing "N" inductors and "N" capacitors is presented. In the case of a nondisipative filter, system behaviour is described by a 2N-th order differential equation (1) where Δu denotes voltage error, e(t) the driving force and ip(t) the load current. Coefficients a_i and b_i depend on the values of inductors and capacitors.

$$\Delta u+\sum_{i=1}^{N}a_{2i}\Delta u^{(2i)}+\sum_{i=1}^{N}a_{2i}u^{*(2i)}+u^* = e-\sum_{i=1}^{N}b_{2i-1}i_p^{(2i-1)} \quad \ldots(1)$$

$$\vec{x}= [x_1,x_2,\ldots x_{2N}]^T=[\Delta u,\Delta\dot{u},\Delta\ddot{u},\ldots \Delta u^{(2N-1)}] \quad \ldots(2)$$

$$\sigma=\sum_{i=1}^{2N}k_i x_i=\sum_{i=1}^{2N}k_i\Delta u^{(i-1)} \;;\; e=-E\,\mathrm{sgn}(\sigma) \quad \ldots(3)$$

The state of the output filter is determined by inductor currents and capacitor charges. Applying the appropriate linear transform, the state vector could be given in the form (2). A linear combination of these states should be formed (equation (3)) and led to a comparator controlling the state of switches. Choosing the Liapunov's function $V(\vec{x})=\sigma^2$ and finding it's time derivative $\dot{V}(\vec{x})$, stability condition $\dot{V}<0$ is found to be met if the DC-bus voltage has sufficient magnitude (eq. (4)). In the case of stable operation, $V=\sigma^2$ decays and reaches zero. So, the motion of the system in 2N-dimensional space is restricted to 2N-1 -dimensional subspace defined by $\sigma=0$. Such a motion is known as the sliding mode. Transient response is now defined by equation (5). From this equation it's clear that the feedback coefficients k1,k2,..etc. completely determine system dynamics. These should be choosen so that the initial mismatch $\Delta u(0)$ exponentially tends to zero. Once the voltage error is eliminated, DC-bus voltage necessary to hold it at zero is reduced to the value given by expression (6). In the case of a second order output filter, the aforementioned switching algorithm can be implemented with minimum circuit complexity. Function $\sigma(\Delta u,\Delta\dot{u})$ is easy to obtain by the measurement of the capacitor

$$E>\left|x_1+\frac{a_{2N}}{k_{2N}}\sum_{i=1}^{2N-1}k_i x_{i+1}-\sum_{i=1}^{N}a_{2i}x_{2i+1}-\sum_{i=1}^{N}b_{2i-1}i_p^{(2i-1)}-u^*-\sum_{i=1}^{N}a_{2i}u^{*(2i)}\right|_{max} \quad \ldots(4)$$

current (Fig. 1). If stability conditions are met, the motion in the state plane ($\Delta u, \Delta\dot{u}$) is restricted to a straight line ($\Delta u + T\Delta\dot{u} = o$). The initial error will decay with time constant (T). Once the error is eliminated, it stays zero under the condition given by expression (7).

$$\sigma = 0 \Rightarrow k_1\Delta u + k_2\Delta\dot{u} + k_3\Delta\ddot{u} \ldots + k_{2N}\Delta u^{(2N-1)} = 0 \quad \ldots(5)$$

$$E > \left| u^* + \sum_{i=1}^{N} a_{2i} u^{*(2i)} + \sum_{i=1}^{N} b_{2i-1} i_p^{(2i-1)} \right|_{max} \quad \ldots(6)$$

$$E > \left| u^* + LC u^{*\prime} + L i_p^{\prime} \right|_{max} \quad \ldots(7)$$

Switching rate moderator. Ideal sliding mode implies an infinite switching rate. Since the commutation frequency of an inverter bridge must be limited and stably set up, some form of switching rate moderator should be employed. As a result of the limited switching rate, the sliding mode is no longer ideal, i.e. system motion in the state plane performs in the vicinity of the line ($\Delta u + T\Delta\dot{u} = o$). Self-oscillating frequency setting can be achieved by introducing a time-delay in the comparator, and this is investigated hereafter. An approximate transfer function of the comparator with time-delay Td is found by the method of harmonic linearization and given by expression (8). It is assumed that the referent waveform is zero ($u^* = o$) and that self-oscillations of amplitude (A) appear at frequency (ω).

$$W_C(s) = \frac{4E}{\pi A}\left[\cos(\omega T_d) - \frac{s}{\omega}\sin(\omega T_d)\right] \quad \ldots(8)$$

$$T_d = \mathrm{arctg}(\omega T)/\omega \quad \ldots(9)$$

If Wl(s) denotes the transfer function of the system linear part, the characteristic polynomial is found as $1+W_c(s)W_l(s)=o$. The self-oscillating mode takes place at frequency (ω) that equals to zero the real parts of the polynomial roots. The necessary delay Td for setting the switching frequency is expressed by equation (9). However, the formula holds only for zero referent waveform. It's shown by McMurray (1) that in the case of a voltage supplied inverter with two-state switching regulator the commutation frequency depends on the square of the modulation index and the switching-to-output frequency ratio is generally not a commensurate number. Inherent randomness in commutation gives rise to beat frequencies and subharmonics in the output voltage spectrum. In order to supress these drawbacks and synchronize the switching instants, additional circuitry must be employed. Significant improvements can be gained by introducing adaptive hysteresis in the comparator, as reported by Kernick et al (2) and Kawamura and Hoft (3).

III BASIC CONCEPT OF LINEAR APPROACH

The linear approach of applying instantaneous feedback in VSI calls for a pulse-width modulator placed in the feedback loop. Sampled data of the referent waveform and the output filter variables serves as the modulation signal that linearly affects the width of the pulses appearing synchronously across the inverter bridge. Setting the sample time in such a way that the sample rate is commensurable with the output frequency, the quality of the spectrum is inherently obtained.

The spectrum is discrete, since the position of the pulses is stationary along the period of the output voltage. The inverter output impedance at relevant frequencies is a measure of sensitivity to load injected harmonics and is expected to decrease by enlarging the voltage-error gain. In order to derive gain limits, stability conditions of such a discrete-time system must be taken into account. The basic circuit for VSI with a second order filter is given in Fig. 1 as controller type (B). Important system parameters are listed below :

- T -the sample time
- ω -output filter resonant frequency
- Z -output filter characteristic impedance
- R -capacitor current feedback coefficient
- A -pulse width modulator saturation level
- G =E/A linearized modulator gain

The modulator sets the pulse width (τ_k) according to the modulation signal samples. The driving force e(t) could be described by expression (1o) :

$$U_m(kT) = U^*(kT) - U(kT) - R i_C(kT)$$

$$e(t) = \begin{cases} E \cdot \mathrm{sgn}(U_m(kT)) & ;\ 0 \leq t - kT < \tau_k \\ 0 & ;\ \tau_k < t - kT < T \end{cases} \quad \ldots(1o)$$

$$\tau_k = \begin{cases} T & ;\ |U_m(kT)/A| > 1 \\ T \cdot |U_m(kT)/A| & ;\ |U_m(kT)/A| \leq 1 \end{cases}$$

The matrix differential equation (11) describing an unloaded inverter can be translated into difference form (12). $\Phi(t)$ is the system fundamental matrix (equation (13)).

$$\dot{\vec{x}} = [A]\vec{x} + \vec{b}e(t)\ ;\ \vec{x} = [u, i_c]^T ;\ \vec{b} = [0, 1/L] \ldots(11)$$

$$\vec{X}_{k+1} = \Phi(T)\vec{X}_k + E\,\mathrm{sgn}(U_m(kT))\Phi(T)\int_0^{\tau_k}\Phi(-t)\vec{b}\,dt \quad \ldots(12)$$

$$\Phi(t) = \exp([A]t) = \begin{bmatrix} \cos(\omega t) & Z\sin(\omega t) \\ -\sin(\omega t)/Z & \cos(\omega t) \end{bmatrix} \quad \ldots(13)$$

Introducing new state variables (equation(14)) state transition is defined by (15). Matrix [F] is constant while [D] has variable coefficients.

$$\vec{y} = [S]\vec{x} = [u + Ri_c, i_c]^T ;\ [S] = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \quad \ldots(14)$$

$$\vec{y}_{k+1} = [F]\vec{y}_k - E[D]\vec{y}_k\ ;\ [F] = [S]^{-1}[D][S] \quad \ldots(15)$$

$$[D] = \begin{bmatrix} 2\sin\frac{\omega\tau_k}{2}\left(\sin\frac{\omega(2T-\tau_k)}{2} + \frac{R}{Z}\cos\frac{\omega(2T-\tau_k)}{2}\right)/|y_1| & ,0 \\ \frac{2}{Z}\sin\frac{\omega\tau_k}{2}\cos\frac{\omega(2T-\tau_k)}{2}/|y_1| & ,0 \end{bmatrix}$$

Equation (15) can be linearized for $y_1 \approx o$, by replacing variable coefficiens $d_{11}(y_1)$ and $d_{21}(y_1)$ with values $d_{11}(o)$ and $d_{21}(o)$. The transfer function $W(z)=y_1(z)/U_m(z)$ for the linearized system is given by the equation (16). Behaviour of the system is predicted by the roots of the characteristic polynomial $1+W(z)=o$. Maximum gain allowable concerning stability is given by expression (17).

$$W(z) = \frac{E}{A}\,\frac{(\sin(\omega T) + \cos(\omega T)R/Z)z - R/Z}{z^2 - 2z\cos(\omega T) + 1}\,\omega T \ldots(16)$$

$$G_{max} = \frac{2(1+\cos(\omega T))}{\omega T(R/Z+R/Z\cos(\omega T)+\sin(\omega T))} \quad \ldots(17)$$

Stability analysis for large disturbances is to be performed by the Liapunov's second method for discrete-time systems, as explained by Murphy and Wu (4). The results of such an analysis show that the gain limit for global stability is slightly below the corresponding value for the linearized system. Hence, formula (17) makes a good approximation for inverter design. If the filter and the switching frequency are defined, controller design consists in setting the parameters (G) and (R) in such a manner that sufficient system damping and a large enaugh stability margin are obtained. Global stability and the damping $\xi > 0.6$ are obtained employing the parameter values according to the expression :

$$G=0.5(G\ max) \quad ; \quad R=1.2\cdot Z/\sqrt[2]{G} \quad \ldots(18)$$

IV SIMULATIONS AND EXPERIMENTS

Static and dynamic characteristics of a VSI employing output filter state feedback in a linear and a nonlinear manner are compared on the basis of computer simulations and experimental results. All the results are related to the 5oo W prototype with DC-bus voltage E=4oo V, 22o V 5o Hz output voltage , full switching bridge and second order output filter with L=3o mH and C=1o μf. Nonlinear controller parameters are Td=75 μs and T=1 ms (see Fig. 1, controller type A). Discrete -time regulator parameters are set according to (18) with T=35o μs taken as the sample time (Fig. 1, controller type B). The prototype output voltage waveform is recorded for both controllers under no-load condition. Discrete Fourier Transform of the recorded waveforms is given in Fig. 2. Load-step response oscillograms are presented in Fig. 3. For the purpose of inverter output impedance measurement, computer simulations are performed. In the simulation model, referent input u* is set to zero, load current harmonics injected and their influence on the output voltage observed.

$$i_p(t)=0.1\cdot Inom(\sin 3\omega t+\sin 5\omega t+\sin 7\omega t) \ldots(19)$$

$$Z_{k-th}=U_{k-th}/0.1\cdot Inom \quad ; \quad k=3,5,7 \quad ..(20)$$

The hypothetical load current is expressed by equation (19) . The magnitude of injected harmonics is set to 1o% of the nominal load current. The values of impedances at relevant frequencies are found according to (2o) and given in Table 1 for both controllers. The lower inverter impedance means higher tolerance level to load injected harmonics.

TABLE 1 - Comparison of controller A and B in reduction of output impedance.

controller type	Z_3(%)	Z_5(%)	Z_7(%)
A			
T=1.5 ms	o.31	o.5	o.43
T=1.o ms	o.22	o.48	o.42
T=o.5 ms	o.17	o.47	o.42
B			
G=o	39	187	15o
G=4	4.o3	5.94	7.73
G=16	2.96	3.o2	3.o5

V CONCLUSION

Implementation of instantaneous feedback in VSI is proven to be high insensitive to nonlinear loading and ensures inherently stable output voltage magnitude and phase. Stability analysis and synthesis methods are performed for both the linear and nonlinear way of applying the output filter state feedback. Simulations and experiments carried out confirm that both approaches supress undershoot voltage transients and completely eliminate output filter resonant phenomena. Nonlinear controller is more efficient in the inverter output impedance reduction. Spectral analysis of the current and voltage waveforms show that the linear controller reduces the output impedance at relevant frequencies to 3% of the nominal impedance, while for the nonlinear controller this reduction is o.5% , under the same operating conditions. Nonlinear approach employs a minimum of circuit elements and gives time-optimal response along with a robustness to load disturbances, but has the disadvantage of asynchronous switching. Randomness in commutations gives a rise to beat-frequencies and deteriorates the output voltage spectrum. These drawbacks could be avoided by making the controller adaptive (reference (3)). The linear approach, proposed in the present paper entails synchronous pulse-width modulator in the feedback loop and the switching is inherently synchronous. Voltage Source Inverter with this type of controller comprises both superior dynamic performance and a well defined output voltage spectrum.

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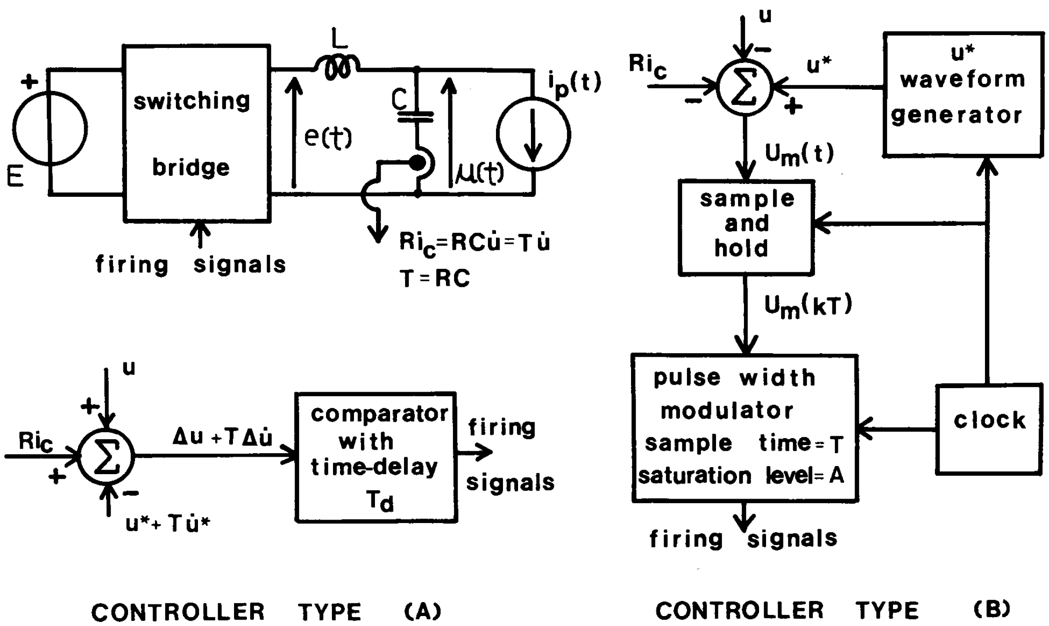

Figure 1 Nonlinear (A) and linear (B) controller of the output voltage instantaneous value

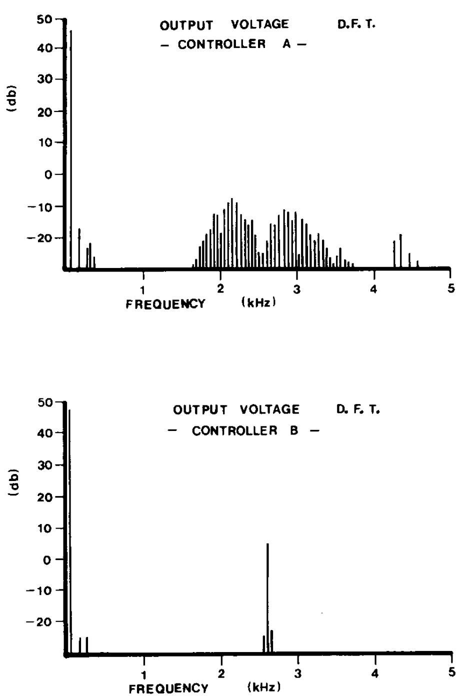

Figure 2 Output voltage discrete spectrum

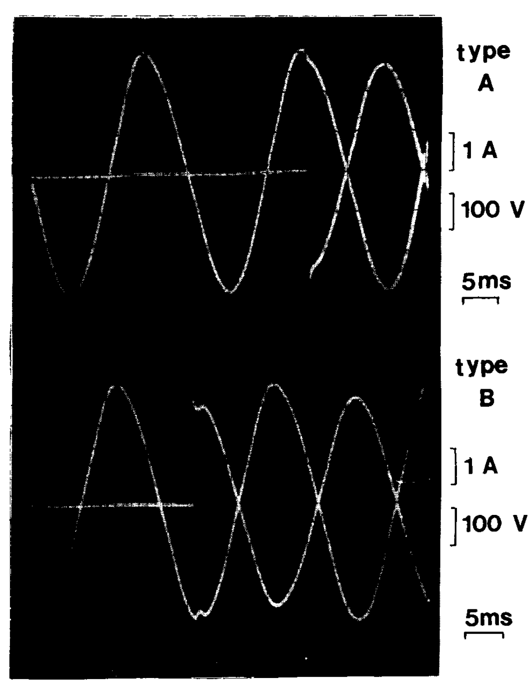

Figure 3 Load step response