

# Efficient Electromagnetic Analysis Using Electrically Large Curved $p$ -Refined Hierarchical Anisotropic Inhomogeneous Finite Elements

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The finite element method (FEM) in its various forms and implementations has been effectively used for solving both open-region (e.g., antenna/scattering) and closed-region (e.g., waveguide/cavity) electromagnetic problems. By its inherent features, the FEM is especially suitable for three-dimensional (3-D) frequency-domain modeling, analysis, and design of electromagnetic structures that contain geometrical and material complexities. Traditional FEM tools are low-order (small-domain or subdomain) techniques – the electromagnetic structure under consideration is modeled by volume geometrical elements that are electrically very small, on the order of  $\lambda/10$  in each dimension,  $\lambda$  being the wavelength in the medium, and with planar sides, and the fields within the elements are approximated by low-order basis functions, which results in very large requirements in computational resources. An alternative is the higher order (large-domain or entire-domain) computational approach, which utilizes higher order basis functions defined on large (e.g., on the order of  $\lambda$  in each dimension) curvilinear geometrical elements. However, although higher order FEM modeling is becoming a mainstream activity in FEM research and practice, there seems to be a lack of investigations and reported results on the actual higher order and large-domain modeling of material complexities and a full exploitation of modeling and computational flexibility, versatility, and efficiency of large curved finite elements with  $p$ -refined high-order field approximations in applications involving arbitrary material anisotropy and inhomogeneity.

This paper presents accurate and efficient solutions in the frequency domain of 3-D open- and closed-region problems in the presence of general anisotropic inhomogeneous electromagnetic materials using higher order large-domain FEM modeling. The solutions implement Lagrange-type generalized curved parametric hexahedral finite elements of arbitrary geometrical-mapping orders for the approximation of geometry in conjunction with higher order curl-conforming hierarchical polynomial vector basis functions of arbitrary field-expansion orders for the approximation of fields within the elements. Elements are generally filled with anisotropic inhomogeneous materials with continuous spatial variations of complex relative permittivity and permeability tensors described by Lagrange interpolation polynomials of arbitrary material-representation orders.

Analysis of open-region scattering structures is performed truncating the FEM domain by a hybridization with a higher order method of moments. Analysis of closed-region microwave waveguide structures introduces a simple single-mode boundary condition across waveguide ports. Examples to be presented demonstrate efficient and accurate simulations of anisotropic continuously inhomogeneous scattering and waveguide structures using large (extending  $2\lambda$  in each dimension) anisotropic inhomogeneous curved finite elements with  $p$ -refined field distributions of high (e.g., seventh) approximation orders.