Direct and Indirect Time-Domain FEM Higher Order Solutions to 3-D Closed-Region Problems

Nada J. Šekeljić* [1], Sanja B. Manić [1], Milan M. Ilić [1],[2], and Branislav M. Notaroš [1]
(1) Electrical & Computer Engineering Department, Colorado State University, Fort Collins, CO
(2) School of Electrical Engineering, University of Belgrade, Serbia
inadasek@engr.colostate.edu, smanic@engr.colostate.edu, milan.ilic@etf.rs,
notaros@colostate.edu

The finite element method (FEM), as one of the most powerful and versatile general numerical tools for electromagnetic-field computations based on discretizing partial differential equations (PDEs), has been especially effectively used for full-wave three-dimensional (3-D) solutions to closed-region (e.g., waveguide) problems in the frequency domain (FD). However, FEM analysis of 3-D waveguide structures in the time domain (TD) is also of great practical importance. TD FEM analysis is effective in modeling of time-varying and nonlinear problems and in broadband characterizations and designs of electromagnetic devices and systems. Nevertheless, a rather disproportionate body of work exists in the published literature in the development, implementation, and application of time-domain FEM methods and techniques, when compared to their frequency-domain counterparts.

This paper presents both direct and indirect TD FEM solutions to 3-D closed-region problems including multiport waveguide structures with arbitrary metallic and dielectric discontinuities. Direct TD FEM modeling is based on a direct numerical discretization of TD PDEs, approximating the spatially and temporally varying electric field by efficient vector basis functions defined on curved hexahedral elements with time-dependent field-distribution coefficients, which are determined using a time-marching procedure, with an implicit unconditionally stable time-stepping finite difference scheme and the Newmark-beta method. Indirect TD FEM modeling is based on the FD FEM analysis of the structure combined with the discrete Fourier transform and its inverse, requiring FEM solutions at many discrete frequency points. Both direct and indirect TD FEM techniques implement Lagrange-type generalized curved parametric hexahedral finite elements of arbitrary geometrical-mapping orders, filled with anisotropic inhomogeneous materials with continuous spatial variations of complex relative permittivity and permeability tensors described by Lagrange interpolation polynomials of arbitrary material-representation orders, and curl-conforming hierarchical polynomial vector basis functions of arbitrary field-expansion orders for the approximation of the electric field vector within the elements.

Numerical examples include 3-D waveguide structures with metallic and penetrable discontinuities of different shapes, composed of homogeneous and continuously inhomogeneous materials. Results demonstrate excellent numerical properties of both direct and indirect TD FEM solutions, and serve to cross-validate the two techniques against each other, and to evaluate their numerical properties, advantages, and deficiencies. Examples also demonstrate using field expansions of very high orders and large-domain TD FEM meshes using a very small number of large conformal curved hexahedral finite elements, which results in solutions with minimal total numbers of unknowns. Finally, the results show excellent stability, convergence, and versatility of higher order spatial basis functions in conjunction with the implemented time-stepping processes in determining the transient field responses across large finite elements.