Trilinear Hexahedral Finite Elements with Higher-Order Polynomial Field Expansions for Hybrid SIE/FE Large-Domain Electromagnetic Modeling

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1. Introduction

Surface integral equation (SIE) formulation in conjunction with the method of moments (MoM) [1, 2] is, in the opinion of the authors of this paper, the "best" choice for modeling of 3D open-region antenna and scattering problems that predominantly consist of metallic surfaces and wires, and include some homogeneous dielectric parts. Partial differential-equation numerical methods, such as the class of finite-element (FE) techniques [3, 4], are extremely efficient at modeling of 3D structures that predominantly consist of arbitrary shaped heterogeneous (isotropic or anisotropic) nonmetallic material sections. The best way to fully benefit from both approaches in most general practical cases is to hybridize them in an appropriate fashion.

This paper presents our on-going research efforts in hybrid SIE/FE modeling. We propose finite elements in the form of electrically large trilinear hexahedrons with higher-order polynomial field expansions. Such FE's are extremely suitable for hybridization with the large-domain SIE Galerkin-type MoM [5], which uses electrically large bilinear quadrilateral boundary surface elements with higher-order polynomial expansions for electric and magnetic surface currents. The efficiency and convergence properties of higher-order polynomial field expansions for FE modeling are clearly demonstrated on a simple one-dimensional example.

The idea in our SIE/FE hybridization is to apply MoM (SIE) everywhere except inside the inhomogeneous sections, which are treated by a FE technique. The MoM and FE solutions are coupled on the boundary surface between the MoM and FE regions through the continuity of tangential fields (as in the pure SIE). Basically, the difference between this hybrid technique and pure MoM is the replacement of the Green's function inside the FE region, which in general case is not known, with the solution of the FE method. The FE solution of Maxwell's differential equations (or Helmoltz equations) inside an inomogeneous object thus actually generates a numerical Green's function for the object.

2. Trilinear Hexahedral Finite Elements with Polynomial Field Expansions

FE method solves for fields and discretizes the volume of the object. As a basic finite element for geometrical modeling we propose a trilinear hexahedron [6]. This is a body determined solely by eight arbitrary points in space, which represent its vertices (Fig.1). It can be described analytically as

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$$\boldsymbol{r}(u,v,w)=\frac{1}{8}\left[\boldsymbol{r}_1(1-u)(1-v)(1-w)+\boldsymbol{r}_2(u+1)(1-v)(1-w)+\ldots+\boldsymbol{r}_8(u+1)(v+1)(w+1)\right]$$
$$=\boldsymbol{r}_c+\boldsymbol{r}_u u+\boldsymbol{r}_v v+\boldsymbol{r}_w w+\boldsymbol{r}_{uv}uv+\boldsymbol{r}_{uw}uw+\boldsymbol{r}_{vw}vw+\boldsymbol{r}_{uvw}uvw, \quad -1\le u,v,w\le 1 \tag{1}$$

$\boldsymbol{r}_1$, $\boldsymbol{r}_2$, …, $\boldsymbol{r}_8$ being the position vectors of the hexahedron vertices. The hexahedron sides are generally curved bilinear quadrilateral surfaces.

It is convenient to initially represent the fields inside every hexahedron in the model as

$$\boldsymbol{E}=\frac{1}{K}\left[E'_{vw}\left(\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}v}\times\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}w}\right)+E'_{wu}\left(\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}w}\times\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}u}\right)+E'_{uv}\left(\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}u}\times\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}v}\right)\right], \quad K=\left(\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}u}\times\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}v}\right)\cdot\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}w}, \tag{2}$$

where $\boldsymbol{r}$ is defined in Eq.(1). The field components are approximated by large-domain 3D polynomial expansions in the coordinates u, v, and w given by

$$E'_{vw}(u,v,w)=\sum_{i=0}^{N_u-1}\sum_{j=0}^{N_v}\sum_{k=0}^{N_w}a_{vwijk}\,u^i P_j(v)P_k(w)\;, \qquad P_j(v)=\begin{cases}1-v, & j=0\\ v+1, & j=1\\ v^j-1, & j\ge 2,\ \text{even}\\ v^j-v, & j\ge 3,\ \text{odd}\end{cases} \tag{3}$$

with analogous expressions for E'_{wu} and E'_{uv}. N_u, N_v, and N_w are the adopted degrees of the polynomial, and a_{vwijk} are unknown complex coefficients to be determined. Note that similar expansions in trilinear hexahedral elements are used in the large-domain volume integral equation (VIE) MoM [6, 7, 5]. Polynomial degrees can be high, so electrically large finite elements can be used (large-domain FE technique). This dramatically reduces the overall number of unknowns for a given problem. The field expansions automatically satisfy continuity boundary conditions for tangential fields on surfaces shared by adjacent hexahedrons in the FE mesh. The expressions for the tangential field components on individual sides of a trilinear hexahedron have exactly the same form as the expression for the approximation of the corresponding surface current density vector component on a bilinear quadrilateral [5, 7] representing that side of the trilinear hexahedron. This is extremely convenient for the hybridization of two methods. It enables direct exact coupling between the FE volume modeling and SIE surface modeling.

3. Efficiency and Convergence of Higher-Order Polynomial Expansions: A Simple Example

In order to evaluate the efficiency, accuracy, and convergence of basis functions in Eq.(3) as applied to FE modeling, a simple Galerkin-type FE routine for one-dimensional analysis of wave propagation through layered media [8] is utilized. Consider an electrically thick continuously inhomogeneous dielectric slab with parameters $\varepsilon_\mathrm{r}=4+(2-\mathrm{j}0.1)(1-z/L)^2$ and $\mu_\mathrm{r}=2-\mathrm{j}0.1$, and thickness $L=20\lambda$, backed by a perfectly conducting plane at z = 0, where λ is the averaged wavelength in dielectric and the z-axis is normal to the conducting plane [4]. Let the slab be illuminated from a vacuum by a plane wave with parallel polarization incident at an angle of 70° with respect to the z-axis. FE results for the reflection coefficient are compared with the analytical solution for a multi-layer approximation of the slab [4].

Fig.2 shows the FE computational time, T, as a function of the relative error of the reflection coefficient, δ, for the small-domain FE analysis (the polynomial order is $N = 1$ and the number of elements, M, varies from 30 to 500) and the large-domain FE analysis ($M = 10$ and N varies from 3 to 8). We observe excellent accuracy and convergence properties of basis functions in Eq.(3), and their high efficiency with regards to the relative computational time. The figure demonstrates great numerical advantages of the large-domain FE's over the small-domain FE's.

Of course, the elements cannot be arbitrarily large, i.e., very large structures have to be subdivided into a number of (large) elements. In other words, for a given (small) number of elements, it is not possible to achieve arbitrarily high accuracy by increasing the polynomial order only. Shown in Fig.3 is a family of curves representing T against δ, where each curve corresponds to a fixed M and variations in T and δ are consequences of the increase of N. It can be observed from the figure that the increase of N beyond a certain value does not yield improved accuracy, that is, M has to be increased as well, simultaneously. Optimal M for any given accuracy and the minimal T is determined by the envelope line in the figure. Based on this, and some other numerical experiments, we have adopted 1.5λ to be the optimal dimension of finite elements and the general limit in the FE procedure beyond which the structure is subdivided into smaller, but still large-domain, optimal elements (note that the corresponding small-domain limit is 0.1λ).

4. Conclusions

In this paper, finite elements in the form of electrically large trilinear hexahedrons with higher-order polynomial field expansions are proposed. Numerical benefits from using higher-order expansions for FE modeling are demonstrated on a simple one-dimensional example. Our current and future work includes full numerical implementation of large-domain hexahedral FE's and their hybridization with the large-domain SIE method.

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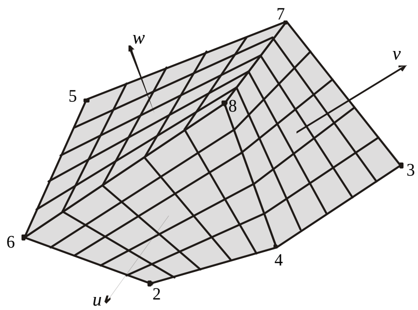

Fig.1. A trilinear hexahedron

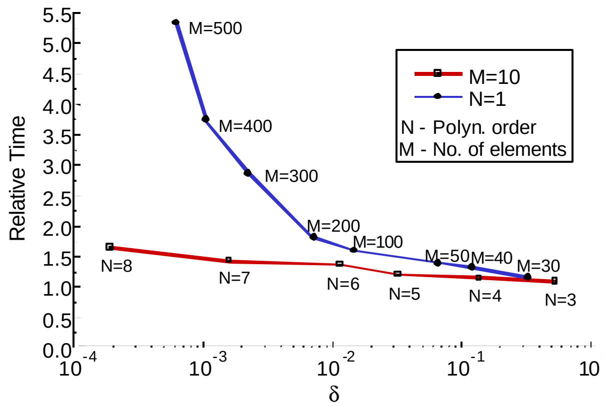

Fig.2. Efficiency analysis: large-domain vs. small-domain FE's (please see text for details).

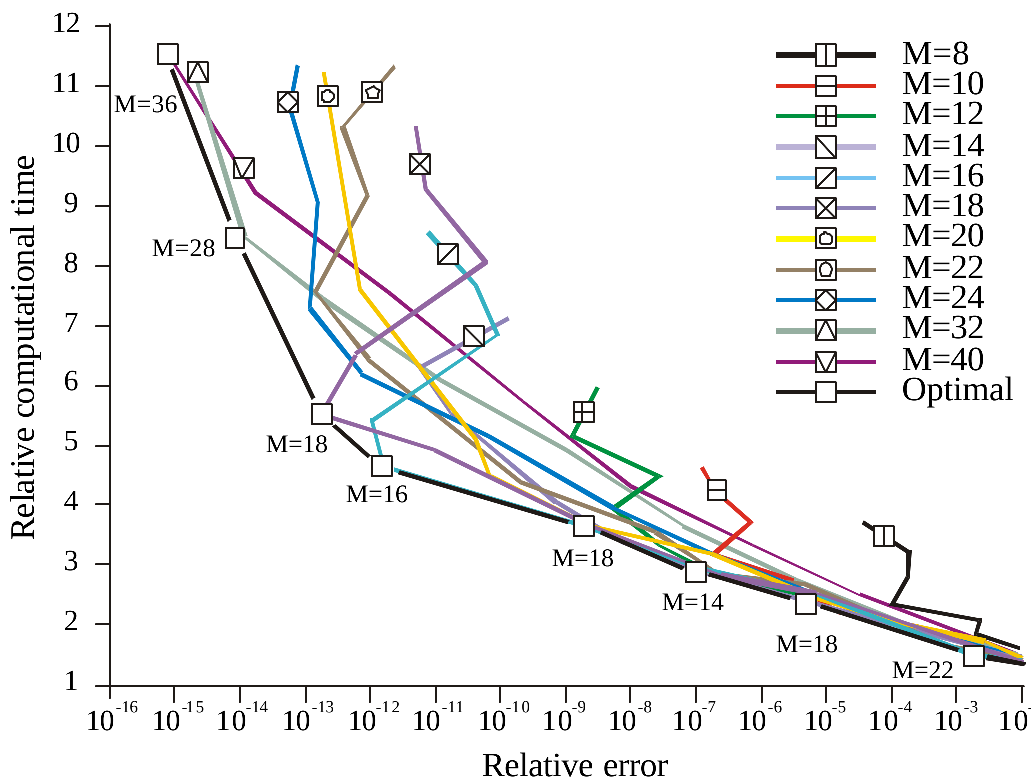

Fig.3. Efficiency analysis: optimally large FE's (please see text for details).